RECOMMENDED PRACTICE
DNV-RP-C202

BUCKLING STRENGTH OF SHELLS

OCTOBER 2002

Since issued in print (October 2002), this booklet has been amended, latest in April 2005.
See the reference to "Amendments and Corrections" on the next page.

DET NORSKE VERITAS

FOREWORD

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ACKNOWLEDGEMENT

This Recommended Practice is developed in close co-operation with the offshore industry, research institutes and universities. All contribution are highly appreciated.

CHANGES

Editorial changes have been made.

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1. Introduction

1.1 Buckling strength of shells

This RP treats the buckling stability of shell structures based on the load and resistance factor design format (LRFD). Chapter 2 gives the stress in closed cylinders. Chapter 3 treats the buckling of circular cylindrical steel shells, see Figure 1.1-1. The shell cylinder may be stiffened by longitudinal stiffeners and/or ring frames.

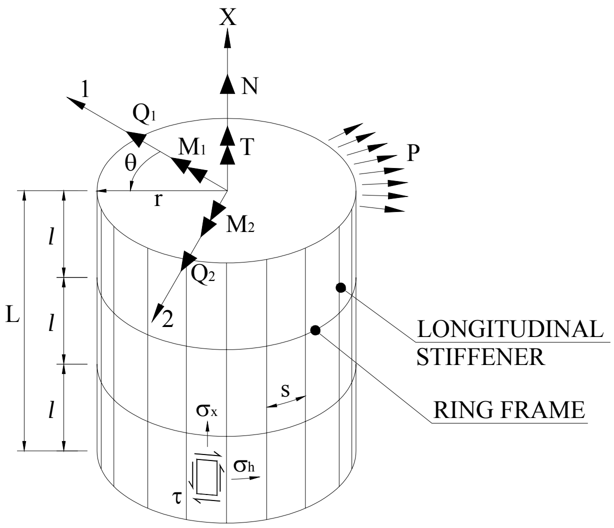


Figure 1.1-1 Stiffened cylindrical shell

It is assumed that the edges are effectively supported by ring frames, bulkheads or end closures.

Stiffened circular cylindrical shells have to be dimensioned against several buckling failure modes. The relevant modes are defined in Section 1.3. To exclude local buckling of longitudinal stiffeners and rings, explicit requirements are given in Section 3.10

In Table 1.3-1 reference is made to recommended methods for buckling analysis with respect to different buckling modes. The methods are to be considered as semi-empirical. The reason for basing the design on semi-empirical methods is that the agreement between theoretical and experimental buckling loads for some cases has been found to be non-existent. This discrepancy is due to the effect of geometric imperfections and residual stresses in fabricated structures. Actual geometric imperfections and residual stresses do not in general appear as explicit parameters in the expressions for buckling resistance. This means that the methods for buckling analysis are based on an assumed level of imperfections. This level is reflected by the tolerance requirements given in *DNV OS-C401; Fabrication and Testing of Offshore Structures*.

The recommended methods for buckling analyses may be substituted by more refined analyses or model tests taking into account the real boundary conditions, the pre-buckling edge disturbances, the actual geometric imperfections, the non-linear material behaviour, and the residual welding stresses.

Chapter 4 treats the buckling of unstiffened conical shells.

1.2 Working Stress Design

This Recommended Practice is written in the load and resistance factor design format (LRFD format) to suit the DNV Offshore Standard DNV-OS-C101. This standard makes use of material- (resistance) and loadfactors as safety factors.
DNV-RP-C202 may be used in combination with working stress design format (WSD) by the following method:
For the formulas used in DNV-RP-C202, including eq. 3.1.3, use a material factor γ_M=1.15. The utilisation checks should be made using a modified permissible usage factor $\eta_p=1.15\beta\eta_0$, see DNV-OS-C201 Sec. 2 Table E1 for η_0 and Sec. 5 Table C1 for β.

1.3 Symbols and Definitions

1.3.1 Symbols

The following symbols are used and may not have a specific definition in the text where they appear:

A	cross-sectional area of a longitudinal stiffener (exclusive of shell flange)
A_c	cross sectional area of complete cylinder section; including longitudinal stiffeners/internal bulkheads if any
A_f	cross sectional area of flange (=bt_f)
A_R	cross-sectional area of a ring frame (exclusive of shell flange)
A_{Req}	required cross sectional area (exclusive of effective plate flange) of ring frame to avoid panel ring buckling
A_w	cross sectional area of web (=ht_w)
C	reduced buckling coefficient
C_1	coefficient
C_2	coefficient
E	Young's modulus = $2.1\cdot10^5$ N/mm^2
G	shear modulus, $G = \frac{E}{2(1+\nu)}$
I	moment of inertia of a longitudinal stiffener (exclusive of shell flange)
I_c	moment of inertia of the complete cylinder section (about weakest axis), including longitudinal stiffeners/internal bulkheads if any

I_{po} polar moment of inertia

I_R effective moment of inertia of a ring frame

I_{sef} moment of inertia of longitudinal stiffener including effective shell width s_e

I_t stiffener torsional moment of inertia (St. Venant torsion).

I_z moment of inertia of a stiffeners neutral axis normal to the plane of the plate

I_h minimum required moment of inertia of ringframes inclusive effective shell flange in a cylindrical shell subjected to external lateral or hydrostatic pressure

I_x minimum required moment of inertia of ringframes inclusive effective shell flange in a cylindrical shell subjected to axial and/or bending

I_{xh} minimum required moment of inertia of ringframes inclusive effective shell flange in a cylindrical shell subjected to torsion and/or shear

L distance between effective supports of the ring stiffened cylinder

L_c total cylinder length

L_H equivalent cylinder length for heavy ring frame

M_{Sd} design bending moment

$M_{1,Sd}$ design bending moment about principal axis 1

$M_{2,Sd}$ design bending moment about principal axis 2

N_{Sd} design axial force

Q_{Sd} design shear force

$Q_{1,Sd}$ design shear force in direction of principal axis 1

$Q_{2,Sd}$ design shear force in direction of principal axis 2

T_{Sd} design torsional moment

$Z_L = \frac{L^2}{rt}\sqrt{1-\nu^2}$, curvature parameter

$Z_l = \frac{l^2}{rt}\sqrt{1-\nu^2}$, curvature parameter

$Z_s = \frac{s^2}{rt}\sqrt{1-\nu^2}$, curvature parameter

a Factor

b flange width, factor

b_f flange outstand

c Factor

e distance from shell to centroid of ring frame exclusive of any shell flange

e_f flange eccentricity

f_{ak} reduced characteristic buckling strength

f_{akd} design local buckling strength

f_E elastic buckling strength

f_{Ea} elastic buckling strength for axial force.

f_{Eh} elastic buckling strength for hydrostatic pressure, lateral pressure and circumferential compression.

f_{Em} elastic buckling strength for bending moment.

f_{ET} elastic buckling strength for torsion.

$f_{E\tau}$ elastic buckling strength for shear force.

f_k characteristic buckling strength

f_{kc} characteristic column buckling strength

f_{kcd} design column buckling strength

f_{ks} characteristic buckling strength of a shell

f_{ksd} design buckling strength of a shell

f_r characteristic material strength

f_T torsional buckling strength

f_y yield strength of the material

h web height

h_s distance from stiffener toe (connection between stiffener and plate) to the shear centre of the stiffener.

i radius of gyration

i_c radius of gyration of cylinder section

i_h effective radius of gyration of ring frame inclusive affective shell flange

k effective length factor, column buckling

l distance between ring frames

l_e equivalent length

l_{ef} effective width of shell plating

l_{eo} equivalent length

l_T torsional buckling length

p_{Sd} design lateral pressure

r shell radius

r_e equivalent radius

r_f radius of the shell measured to the ring flange

r_r radius (variable)

r_0 radius of the shell measured to the neutral axis of ring frame with effective shell flange, l_{eo}

s distance between longitudinal stiffeners

s_e effective shell width

t shell thickness

t_b thickness of bulkhead

t_e equivalent thickness

t_f thickness of flange

t_w	thickness of web
w	initial out-of roundness
z_t	distance from outer edge of ring flange to centroid of stiffener inclusive effective shell plating
α, α_A	coefficients
α_B, α_C	coefficients
β	coefficient
δ_0	initial out-of-roundness parameter
γ_M	material factor
η	coefficient
$\bar{\lambda}$	reduced column slenderness
$\bar{\lambda}_s$	reduced shell slenderness
$\bar{\lambda}_T$	reduced torsional slenderness
μ	Coefficient
θ	circumferential co-ordinate measured from axis 1
ρ	Coefficient
ν	Poisson's ratio = 0.3
$\sigma_{a,Sd}$	design membrane stress in the longitudinal direction due to uniform axial force
$\sigma_{h,Sd}$	design membrane stress in the circumferential direction
$\sigma_{hR,Sd}$	design membrane stress in a ring frame
$\sigma_{hm,Sd}$	design circumferential bending stress in a shell at a bulkhead or a ringframe
$\sigma_{j,Sd}$	design equivalent von Mises' stress
$\sigma_{m,Sd}$	design membrane stress in the longitudinal direction due to global bending
$\sigma_{x,Sd}$	design membrane stress in the longitudinal direction
$\sigma_{xm,Sd}$	design longitudinal bending stress in a shell at a bulkhead or a ringframe
τ_{Sd}	design shear stress tangential to the shell surface (in sections x = constant and θ = constant)
$\tau_{T,Sd}$	design shear stress tangential to the shell surface due to torsional moment
$\tau_{Q,Sd}$	design shear stress tangential to the shell surface due to overall shear forces
ξ	coefficient
ψ	coefficient
ζ	coefficient

1.3.2 Definitions

A general ring frame cross section is shown Figure 1.2-1,

A	Centroid of ring frame with effective shell flange, l_{eo}
B	Centroid of ring frame exclusive any shell flange
C	Centroid of free flange

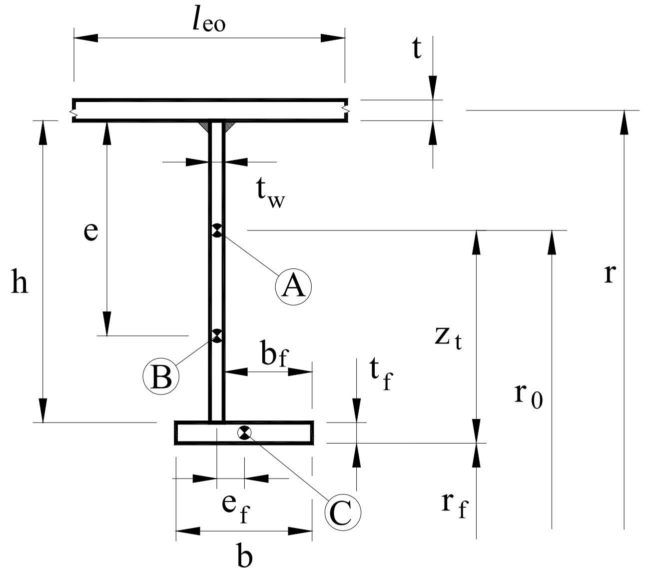


Figure 1.3-1 Cross sectional parameters for a ring frame

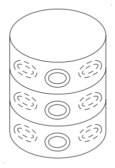
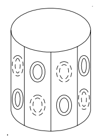
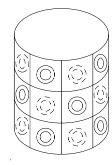
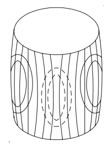
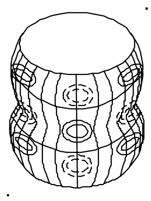
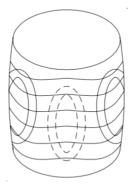
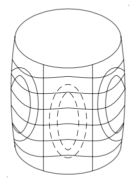
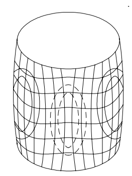
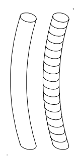
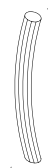
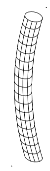
1.4 Buckling modes

The buckling modes for stiffened cylindrical shells are categorised as follows:

a) Shell buckling: Buckling of shell plating between rings/ longitudinal stiffeners.
b) Panel stiffener buckling: Buckling of shell plating including longitudinal stiffeners. Rings are nodal lines.
c) Panel ring buckling: Buckling of shell plating including rings. Longitudinal stiffeners act as nodal lines.
d) General buckling: Buckling of shell plating including longitudinal stiffeners and rings.
e) Column buckling: Buckling of the cylinder as a column.
For long cylindrical shells it is possible that interaction between local buckling and overall column buckling may occur because second order effects of axial compression alter the stress distribution calculated from linear theory. It is then necessary to take this effect into account in the column buckling analysis. This is done by basing the column buckling on a reduced yield strength, f_{kc}, as given for the relevant type of structure.
f) Local buckling of longitudinal stiffeners and rings. Section 3.10

The buckling modes and their relevance for the different cylinder geometries are illustrated in Table 1.3-1

Table 1.4-1 Buckling modes for different types of cylinders

Buckling mode	*Type of structure geometry*		
	Ring stiffened (unstiffened circular)	*Longitudinal stiffened*	*Orthogonally stiffened*
a) Shell buckling	 Section 3.4	 Section 3.3	 Section 3.3
b) Panel stiffener buckling		 Section 3.6	 Section 3.7
c) Panel ring buckling	 Section 3.5		 Section 3.7
d) General buckling			 Section 3.7
e) Column buckling	 Section 3.8	 Section 3.8	 Section 3.8

2. Stresses in Closed Cylinders

2.1 General

The stress resultants governing the stresses in a cylindrical shell is normally defined by the following quantities:

N_{Sd} = Design axial force
M_{Sd} = Design bending moments
T_{Sd} = Design torsional moment
Q_{Sd} = Design shear force
p_{Sd} = Design lateral pressure

Any of the above quantities may be a function of the axial co-ordinate x. In addition p_{Sd} may be a function of the circumferential co-ordinate θ, measured from axis 1. p_{Sd} is always to be taken as the difference between internal and external pressures, i.e. p_{Sd} is taken positive outwards.

Actual combinations of the above actions are to be considered in the buckling strength assessments.

2.2 Stresses

2.2.1 General

The membrane stresses at an arbitrary point of the shell plating, due to any or all of the above five actions, are completely defined by the following three stress components:

$\sigma_{x,Sd}$ = design membrane stress in the longitudinal direction (tension is positive)
$\sigma_{h,Sd}$ = design membrane stress in the circumferential direction (tension is positive)
τ_{Sd} = design shear stress tangential to the shell surface (in sections x = constant and θ = constant)

2.2.2 Longitudinal membrane stress

If the simple beam theory is applicable, the design longitudinal membrane stress may be taken as:

$$\sigma_{x,Sd} = \sigma_{a,Sd} + \sigma_{m,Sd} \tag{2.2.1}$$

where $\sigma_{a,Sd}$ is due to uniform axial force and $\sigma_{m,Sd}$ is due to bending.

For a cylindrical shell without longitudinal stiffeners:

$$\sigma_{a,Sd} = \frac{N_{Sd}}{2\pi r t} \tag{2.2.2}$$

$$\sigma_{m,Sd} = \frac{M_{1,Sd}}{\pi r^2 t}\sin\theta - \frac{M_{2,Sd}}{\pi r^2 t}\cos\theta \tag{2.2.3}$$

For a cylindrical shell with longitudinal stiffeners it is usually permissible to replace the shell thickness by the equivalent thickness for calculation of longitudinal membrane stress only:

$$t_e = t + \frac{A}{s} \tag{2.2.4}$$

2.2.3 Shear stresses

If simple beam theory is applicable, the membrane shear stress may be taken as:

$$\tau_{Sd} = \left|\tau_{T,Sd} + \tau_{Q,Sd}\right| \tag{2.2.5}$$

where $\tau_{T,Sd}$ is due to the torsional moment and $\tau_{Q,Sd}$ is due to the overall shear forces.

$$\tau_{T,Sd} = \frac{T_{Sd}}{2\pi r^2 t} \tag{2.2.6}$$

$$\tau_{Q,Sd} = -\frac{Q_{1,Sd}}{\pi r t}\sin\theta + \frac{Q_{2,Sd}}{\pi r t}\cos\theta \tag{2.2.7}$$

where the signs of the torsional moment and the shear forces must be reflected. Circumferential and longitudinal stiffeners are normally not considered to affect τ_{Sd}.

2.2.4 Circumferential membrane stress

For an unstiffened cylinder the circumferential membrane stress may be taken as:

$$\sigma_{h,Sd} = \frac{p_{Sd}\, r}{t} \tag{2.2.8}$$

provided p_{Sd} is constant (gas pressure) or a sine or cosine function of θ (liquid pressure).

For a ringstiffened cylinder (without longitudinal stiffeners) the circumferential membrane stress midway between two ring frames may be taken as:

$$\sigma_{h,Sd} = \frac{p_{Sd}\, r}{t} - \frac{\alpha\zeta}{\alpha+1}\left(\frac{p_{Sd}\, r}{t} - \nu\sigma_{x,Sd}\right) \tag{2.2.9}$$

where

$$\zeta = 2\frac{\sinh\beta\cos\beta + \cosh\beta\sin\beta}{\sinh 2\beta + \sin 2\beta}, \quad \text{but } \zeta \geq 0 \tag{2.2.10}$$

$$\beta = \frac{l}{1.56\sqrt{rt}} \tag{2.2.11}$$

$$\alpha = \frac{A_R}{l_{eo}\, t} \tag{2.2.12}$$

$$l_{eo} = \frac{l}{\beta}\left(\frac{\text{Cosh } 2\beta - \cos 2\beta}{\text{Sinh } 2\beta + \sin 2\beta}\right) \quad \textbf{(2.2.13)}$$

ζ and l_{eo} may also be obtained from Figure 2.2-1.

For simplification of the analysis the following approximation may be made:

$l_{eo} = l$ or $l_{eo} = 1.56\sqrt{r\,t}$ whichever is the smaller.

For the particular case when p_{Sd} is constant and $\sigma_{x,Sd}$ is due to the end pressure alone, the above formula may be written as:

$$\sigma_{h,Sd} = \frac{p_{Sd}\,r}{t}\left(1 - \frac{\alpha\left(1-\frac{\nu}{2}\right)\zeta}{\alpha+1}\right) \quad \textbf{(2.2.14)}$$

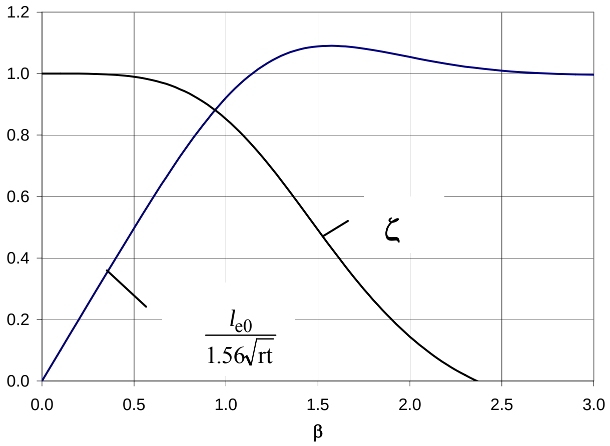


Figure 2.2-1 The parameters l_{eo} and ζ

2.2.5 Circumferential stress in a ring frame

For ring stiffened shells the circumferential stress in a ring frame at the distance r_r (r_r is variable, $r_r = r_f$ at ring flange position and $r_r = r$ at shell) from the cylinder axis may be taken as:

$$\sigma_{hR,Sd} = \left(\frac{p_{Sd}\,r}{t} - \nu\sigma_{x,Sd}\right)\frac{1}{1+\alpha}\left(\frac{r}{r_r}\right) \quad \textbf{(2.2.15)}$$

For the particular case when p_{Sd} is constant and $\sigma_{x,Sd}$ is due to the end pressure alone, the above formula can be written as:

$$\sigma_{hR,Sd} = \frac{p_{Sd}\,r}{t}\left(\frac{1-\frac{\nu}{2}}{1+\alpha}\right)\frac{r}{r_r} \quad \textbf{(2.2.16)}$$

For longitudinally stiffened shells α should be replaced by $\frac{A_R}{l\,t}$ in eq. (2.2.15) and (2.2.16).

2.2.6 Stresses in shells at bulkheads and ring stiffeners

2.2.6.1 General

The below stresses may be applied in a check for local yielding in the material based on a von Mises' equivalent stress criterion. The bending stresses should also be accounted for in the fatigue check, but may be neglected in the evaluation of buckling stability.

2.2.6.2 Circumferential membrane stress

The circumferential membrane stress at a ring frame for a ring stiffened cylinder (without longitudinal stiffeners) may be taken as:

$$\sigma_{h,Sd} = \left(\frac{p_{Sd}\,r}{t} - \nu\sigma_{x,Sd}\right)\frac{1}{1+\alpha} + \nu\sigma_{x,Sd} \quad \textbf{(2.2.17)}$$

In the case of a bulkhead instead of a ring, A_R is taken as $\frac{r\,t_b}{(1-\nu)}$, where t_b is the thickness of the bulkhead. For the particular case when p_{Sd} is constant and $\sigma_{x,Sd}$ is due to the end pressure alone, the above formula can be written as:

$$\sigma_{h,Sd} = \frac{p_{Sd}\,r}{t}\left(\frac{1-\frac{\nu}{2}}{1+\alpha} + \frac{\nu}{2}\right) \quad \textbf{(2.2.18)}$$

2.2.6.3 Bending stress

Bending stresses and associated shear stresses will occur in the vicinity of "discontinuities" such as bulkheads and frames. The longitudinal bending stress in the shell at a bulkhead or a ring frame may be taken as:

$$\sigma_{xm,Sd} = \left(\frac{p_{Sd}\,r}{t} - \sigma_{h,Sd}\right)\sqrt{\frac{3}{1-\nu^2}} \quad \textbf{(2.2.19)}$$

where $\sigma_{h,Sd}$ is given in (2.2.17) or (2.2.18).

The circumferential bending stress in the shell at a bulkhead or a ring frame is:

$$\sigma_{hm,Sd} = \nu\sigma_{xm,Sd} \quad \textbf{(2.2.20)}$$

3. Buckling Resistance of Cylindrical Shells

3.1 Stability requirement

The stability requirement for shells subjected to one or more of the following components:

- axial compression or tension
- bending
- circumferential compression or tension
- torsion
- shear

is given by:

$$\sigma_{j,Sd} \le f_{ksd} \tag{3.1.1}$$

$\sigma_{j,Sd}$ is defined in Section 3.2, and the design shell buckling strength is defined as:

$$f_{ksd} = \frac{f_{ks}}{\gamma_M} \tag{3.1.2}$$

The characteristic buckling strength, f_{ks}, is calculated in accordance with Section 3.2.

The material factor, γ_M, is given as:

$$\begin{aligned} \gamma_M &= 1.15 && \text{for } \bar{\lambda}_s < 0.5 \\ \gamma_M &= 0.85 + 0.60\bar{\lambda}_s && \text{for } 0.5 \le \bar{\lambda}_s \le 1.0 \\ \gamma_M &= 1.45 && \text{for } \bar{\lambda}_s > 1.0 \end{aligned} \tag{3.1.3}$$

Shell structures may be subjected to global column buckling. Evaluation of global column buckling is found in Section 3.8.

3.2 Characteristic buckling strength of shells

The characteristic buckling strength of shells is defined as:

$$f_{ks} = \frac{f_y}{\sqrt{1 + \bar{\lambda}_s^4}} \tag{3.2.1}$$

where

$$\bar{\lambda}_s^2 = \frac{f_y}{\sigma_{j,Sd}}\left[\frac{\sigma_{a0,Sd}}{f_{Ea}} + \frac{\sigma_{m0,Sd}}{f_{Em}} + \frac{\sigma_{h0,Sd}}{f_{Eh}} + \frac{\tau_{Sd}}{f_{E\tau}}\right] \tag{3.2.2}$$

$$\sigma_{j,Sd} = \sqrt{(\sigma_{a,Sd} + \sigma_{m,Sd})^2 - (\sigma_{a,Sd} + \sigma_{m,Sd})\sigma_{h,Sd} + \sigma_{h,Sd}^2 + 3\tau_{Sd}^2} \tag{3.2.3}$$

$$\sigma_{a0,Sd} = \begin{cases} 0 & \text{if } \sigma_{a,Sd} \ge 0 \\ -\sigma_{a,Sd} & \text{if } \sigma_{a,Sd} < 0 \end{cases} \tag{3.2.4}$$

$$\sigma_{m0,Sd} = \begin{cases} 0 & \text{if } \sigma_{m,Sd} \ge 0 \\ -\sigma_{m,Sd} & \text{if } \sigma_{m,Sd} < 0 \end{cases} \tag{3.2.5}$$

$$\sigma_{h0,Sd} = \begin{cases} 0 & \text{if } \sigma_{h,Sd} \ge 0, \text{ internal net pressure} \\ -\sigma_{h,Sd} & \text{if } \sigma_{h,Sd} < 0, \text{ ext. net pressure} \end{cases} \tag{3.2.6}$$

$\sigma_{a,Sd}$ = design axial stress in the shell due to axial forces (tension positive), see eq. (2.2.2)

$\sigma_{m,Sd}$ = design bending stress in the shell due to global bending moment (tension positive), see eq. (2.2.3).

$\sigma_{h,Sd}$ = design circumferential stress in the shell due to external pressure (tension positive), see eq (2.2.8), (2.2.9), or (2.2.14). For ring stiffened cylinders shall only stresses midway between rings be used.

τ_{Sd} = design shear stress in the shell due to torsional moments and shear force, see eq. (2.2.5).

f_{Ea}, f_{Em}, f_{Eh} and $f_{E\tau}$ are the elastic buckling strengths of curved panels or circular cylindrical shells subjected to axial compression forces, global bending moments, lateral pressure, and torsional moments and/or shear forces respectively, where:

f_{Ea} = elastic buckling strength for axial force.

f_{Em} = elastic buckling strength for bending moment.

f_{Eh} = elastic buckling strength for hydrostatic pressure, lateral pressure and circumferential compression.

$f_{E\tau}$ = elastic buckling strength for torsion and shear force.

These may be calculated in accordance with Section 3.3 to 3.7 taking the appropriate buckling coefficients into account.

3.3 Elastic buckling strength of unstiffened curved panels

3.3.1 General

This section deals with buckling of shell plate between stiffeners.

The buckling mode to be checked is:

a) Shell buckling, see Section 3.3.2.

3.3.2 Shell buckling

The characteristic buckling strength is calculated from Section 3.2.

The elastic buckling strength of curved panels with aspect ratio *l*/s > 1 is given by:

$$f_E = C\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{s}\right)^2 \quad \textbf{(3.3.1)}$$

A curved panel with aspect ratio *l*/s < 1 may be considered as an unstiffened circular cylindrical shell with length equal to *l*, see Section 3.4.2.

The reduced buckling coefficient may be calculated as:

$$C = \psi\sqrt{1+\left(\frac{\rho\xi}{\psi}\right)^2} \quad \textbf{(3.3.2)}$$

The values for ψ, ξ and ρ are given in Table 3.3-1 for the most important load cases.

Table 3.3-1 Buckling coefficient for unstiffened curved panels, mode a) Shell buckling

	ψ	ξ	ρ
Axial stress	4	$0.702\ Z_s$	$0.5\left(1+\frac{r}{150t}\right)^{-0.5}$
Shear stress	$5.34+4\left(\frac{s}{l}\right)^2$	$0.856\sqrt{\frac{s}{l}}\ Z_s^{3/4}$	0.6
Circumferential compression	$\left[1+\left(\frac{s}{l}\right)^2\right]^2$	$1.04\frac{s}{l}\sqrt{Z_s}$	0.6

The curvature parameter Z_s is defined as:

$$Z_s = \frac{s^2}{rt}\sqrt{1-\nu^2} \quad \textbf{(3.3.3)}$$

3.4 Elastic buckling strength of unstiffened circular cylinders

3.4.1 General

The buckling modes to be checked are:

a) Shell buckling, see Section 3.4.2.
b) Column buckling, see Section 3.8.

3.4.2 Shell buckling

The characteristic buckling strength of unstiffened circular cylinders is calculated from Section 3.2. The elastic buckling strength of an unstiffened circular cylindrical shell is given by:

$$f_E = C\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{l}\right)^2 \quad \textbf{(3.4.1)}$$

The reduced buckling coefficient may be calculated as:

$$C = \psi\sqrt{1+\left(\frac{\rho\xi}{\psi}\right)^2} \quad \textbf{(3.4.2)}$$

The values for ψ, ξ and ρ are given in Table 3.4-1 for the most important load cases.

The curvature parameter Z is defined as:

$$Z_l = \frac{l^2}{rt}\sqrt{1-\nu^2} \quad \textbf{(3.4.3)}$$

For long cylinders the solutions in Table 3.4-1 will be pessimistic. Alternative solutions are:

- Torsion and shear force

If $\frac{l}{r} > 3,85\sqrt{\frac{r}{t}}$ then the elastic buckling strength may be calculated as:

$$f_{E\tau} = 0,25E\left(\frac{t}{r}\right)^{3/2} \quad \textbf{(3.4.4)}$$

- Lateral/hydrostatic pressure

If $\frac{l}{r} > 2,25\sqrt{\frac{r}{t}}$ then the elastic buckling strength may be calculated as:

$$f_{Eh} = 0,25E\left(\frac{t}{r}\right)^2 \quad \textbf{(3.4.5)}$$

Table 3.4-1 Buckling coefficients for unstiffened cylindrical shells, mode a) Shell buckling

	ψ	ξ	ρ
Axial stress	1	$0.702\ Z_l$	$0.5\left(1+\frac{r}{150t}\right)^{-0.5}$
Bending	1	$0.702\ Z_l$	$0.5\left(1+\frac{r}{300t}\right)^{-0.5}$
Torsion and shear force	5.34	$0.856\ Z_l^{3/4}$	0.6
Lateral pressure[1)]	4	$1.04\sqrt{Z_l}$	0.6
Hydrostatic pressure[2)]	2	$1.04\sqrt{Z_l}$	0.6

NOTE 1: Lateral pressure is used when the capped end axial force due to hydrostatic pressure is not included in the axial force.

NOTE 2: Hydrostatic pressure is used when the capped end axial force due to hydrostatic pressure is included in the axial force.

3.5 Ring stiffened shells

3.5.1 General

The buckling modes to be checked are:

a) Shell buckling, see Section 3.4.2.
b) Panel ring buckling, see Section 3.5.2.
e) Column buckling, see Section 3.8.

3.5.2 Panel ring buckling

The rings will normally be proportioned to avoid the panel ring buckling mode. This is ensured if the following requirements are satisfied.

3.5.2.1 Cross sectional area.

The cross sectional area of a ring frame (exclusive of effective shell plate flange) should not be less than A_{Req}, which is defined by:

$$A_{Req} \geq \left(\frac{2}{Z_l^2} + 0.06\right) l\, t \tag{3.5.1}$$

3.5.2.2 Moment of inertia

The effective moment of inertia of a ring frame (inclusive effective shell plate flange) should not be less than I_R, which is defined by:

$$I_R = I_x + I_{xh} + I_h \tag{3.5.2}$$

I_x, I_{xh} and I_h are defined in eq.(3.5.5), (3.5.7) and (3.5.8), (see also Sec. 3.5.2.7), the effective width of the shell plate flange is defined in Sec. 3.5.2.3.

3.5.2.3 Effective width

The effective width of the shell plating to be included in the actual moment of inertia of a ring frame shall be taken as the smaller of:

$$l_{ef} = \frac{1.56\sqrt{rt}}{1 + 12\frac{t}{r}} \tag{3.5.3}$$

and

$$l_{ef} = l \tag{3.5.4}$$

3.5.2.4 Calculation of I_x

The moment of inertia of ring frames inclusive effective width of shell plate in a cylindrical shell subjected to axial compression and/or bending should not be less than I_x, which is defined by:

$$I_x = \frac{\left|\sigma_{x,Sd}\right| t (1 + \alpha_A) r_0^4}{500\, E\, l} \tag{3.5.5}$$

where

$$\alpha_A = \frac{A}{s\, t} \tag{3.5.6}$$

A = cross sectional area of a longitudinal stiffener.

3.5.2.5 Calculation of I_{xh}

The moment of inertia of ring frames inclusive effective width of shell plate in a cylindrical shell subjected to torsion and/or shear should not be less than I_{xh}, which is defined by:

$$I_{xh} = \left(\frac{\tau_{Sd}}{E}\right)^{8/5} \left(\frac{r_0}{L}\right)^{1/5} L r_0 t\, l \tag{3.5.7}$$

3.5.2.6 Simplified calculation of I_h for external pressure

The moment of inertia of ring frames inclusive effective width of shell plate in a cylindrical shell subjected to external lateral pressure should not be less than I_h, which is conservatively defined by:

$$I_h = \frac{\left|p_{Sd}\right| r\, r_0^2\, l}{3E}\left[1.5 + \frac{3\, E\, z_t\, \delta_0}{r_0^2\left(\frac{f_r}{2} - \left|\sigma_{hR,Sd}\right|\right)}\right] \tag{3.5.8}$$

and

$$\frac{f_r}{2} > \sigma_{hR,Sd}$$

The characteristic material resistance, f_r, shall be taken as:

- For fabricated ring frames:
  $f_r = f_T$
- For cold-formed ring frames:
  $f_r = 0.9 f_T$

The torsional buckling strength, f_T, may be taken equal to the yield strength, f_y, if the following requirements are satisfied:

- Flat bar ring frames:

$$h \leq 0.4\, t_W \sqrt{\frac{E}{f_y}} \tag{3.5.9}$$

- Flanged ring frames ($e_f = 0$, for $e_f \neq 0$ see section 3.10):

$$h \le 1.35 t_w \sqrt{\frac{E}{f_y}} \quad (3.5.10)$$

$$b \ge \frac{7h}{\sqrt{10 + \frac{E}{f_y}\frac{h}{r}}} \quad (3.5.11)$$

Otherwise f_T may be obtained from section 3.9.

z_t is defined in Figure 1.3-1. For $\sigma_{hR,Sd}$ see section 2.2.5 and for p_{Sd} see section 2.1.

The assumed mode of deformation of the ring frame corresponds to ovalization, and the initial out-of-roundness is defined by:

$$w = \delta_0 \cos 2\theta \quad (3.5.12)$$

$$\delta_0 = 0.005\, r \quad (3.5.13)$$

Alternatively the capacity of the ring frame may be assessed from 3.5.2.7.

3.5.2.7 Refined calculation of I_h for external pressure

If a ring stiffened cylinder, or a part of a ring stiffened cylinder, is effectively supported at the ends, the following procedure may be used to calculate required moment of inertia I_h. For design it might be recommended to start with equation (3.5.8) to arrive at an initial geometry. (The reason is that I_h is implicit in the present procedure in equations (3.5.23) and (3.5.27)).

When a ring stiffened cylinder is subjected to external pressure the ring stiffeners should satisfy:

$$|p_{Sd}| \le 0.75 \frac{f_k}{\gamma_M} \frac{t r_f \left(1 + \frac{A_R}{l_{eo} t}\right)}{r^2 \left(1 - \frac{\nu}{2}\right)} \quad (3.5.14)$$

where

p_{Sd}	=	design external pressure
t	=	shell thickness
r_f	=	radius of the shell measured to the ring flange, see Figure 1.2-1.
r	=	shell radius
l_{eo}	=	smaller of $1.56\sqrt{rt}$ and l
A_R	=	cross sectional area of ring stiffener (exclusive shell flange)

f_k is the characteristic buckling strength found from:

$$\frac{f_k}{f_r} = \frac{1 + \mu + \bar{\lambda}^2 - \sqrt{\left(1 + \mu + \bar{\lambda}^2\right)^2 - 4\bar{\lambda}^2}}{2\bar{\lambda}^2} \quad (3.5.15)$$

where

$$\bar{\lambda} = \sqrt{\frac{f_r}{f_E}} \quad (3.5.16)$$

The values for the parameters f_r, f_E and μ may be taken as:

The characteristic material strength, f_r, may be taken equal to the yield strength, f_y, if the following requirements are satisfied:

- Flat bar ring frames:

$$h \le 0.4 t_w \sqrt{\frac{E}{f_y}} \quad (3.5.17)$$

- Flanged ring frames ($e_f = 0$, for $e_f \neq 0$ see section 3.10):

$$h \le 1.35 t_w \sqrt{\frac{E}{f_y}} \quad (3.5.18)$$

$$b \ge \frac{7h}{\sqrt{10 + \frac{E}{f_y}\frac{h}{r}}} \quad (3.5.19)$$

Otherwise f_r should be set to f_T. f_T may be obtained from section 3.9.

$$f_E = C_1 \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{L}\right)^2 \quad (3.5.20)$$

where

$$C_1 = \frac{2(1+\alpha_B)}{1+\alpha}\left(\sqrt{1 + \frac{0.27 Z_L}{\sqrt{1+\alpha_B}}} - \frac{\alpha_B}{1+\alpha_B}\right) \quad (3.5.21)$$

$$Z_L = \frac{L^2}{rt}\sqrt{1-\nu^2} \quad (3.5.22)$$

$$\alpha_B = \frac{12(1-\nu^2) I_h}{l t^3} \quad (3.5.23)$$

$$\alpha = \frac{A_R}{l_{eo} t} \quad (3.5.24)$$

$$\mu = \frac{z_t \delta_0}{i_h^2} \frac{r_f}{r} \frac{l}{l_{eo}} \left(1 - \frac{C_2}{C_1}\right) \frac{1}{1 - \left(\frac{\nu}{2}\right)} \quad (3.5.25)$$

$$\delta_0 = 0.005 r \quad (3.5.26)$$

$$i_h^2 = \frac{I_h}{A_R + l_{eo} t} \quad (3.5.27)$$

z_t = distance from outer edge of ring flange to centroid of stiffener inclusive effective shell plating, see Figure 1.2-1.

$$C_2 = 2\sqrt{1+0.27\,Z_L} \quad \textbf{(3.5.28)}$$

L = distance between effective supports of the ring stiffened cylinder. Effective supports may be:

- End closures, see Figure 3.5-1a.
- Bulkheads, see Figure 3.5-1b.
- Heavy ring frames, see Figure 3.5-1c.

The moment of inertia of a heavy ring frame has to comply with the requirement given in section 3.5.2.2 with I_x, I_{xh} and I_h defined in eq. (3.5.5), (3.5.7) and (3.5.8) and with *l* substituted by L_H, which is defined in Figure 3.5-1d.

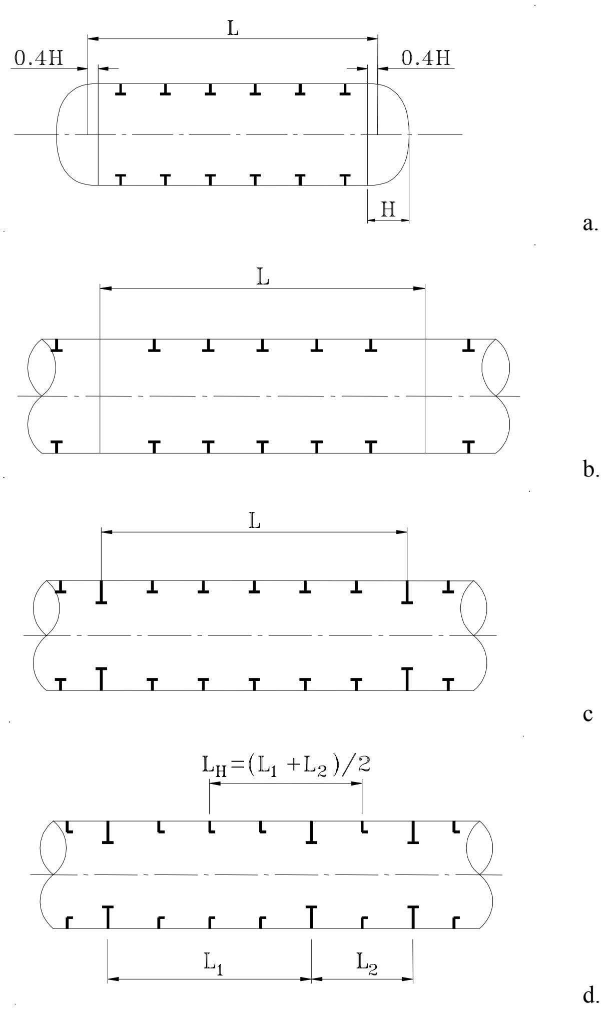


Figure 3.5-1 Definition of parameters L and L_H

3.6 Longitudinally stiffened shells

3.6.1 General

Lightly stiffened shells where $\frac{s}{t} > 3\sqrt{\frac{r}{t}}$ will behave basically as an unstiffened shell and shall be calculated as an unstiffened shell according to the requirements in Section 3.3.2.

Shells with a greater number of stiffeners such that $s/t \le 3\sqrt{r/t}$ may be designed according to the requirements given below or as an equivalent flat plate taking into account the design transverse stress, normally equal to $p_{Sd}\,r/t$.

The buckling modes to be checked are:

a) Shell buckling, see Section 3.6.2
b) Panel stiffener buckling, see Section 3.6.3
e) Column buckling, see Section 3.8.

3.6.2 Shell buckling

The characteristic buckling strength is found from Section 3.2 and the elastic buckling strengths are given in 3.3.2.

3.6.3 Panel stiffener buckling

3.6.3.1 General

The characteristic buckling strength is found from Section 3.2. It is necessary to base the strength assessment on effective shell area. The axial stress $\sigma_{a,Sd}$ and bending stress $\sigma_{m,Sd}$ are per effective shell width, s_e is calculated from 3.6.3.3.

Torsional buckling of longitudinal stiffeners may be excluded as a possible failure mode if the following requirements are fulfilled:

- Flat bar longitudinal stiffeners:

$$h \le 0.4\,t_w\sqrt{\frac{E}{f_y}} \quad \textbf{(3.6.1)}$$

- Flanged longitudinal stiffeners:

$$\overline{\lambda_T} \le 0.6 \quad \textbf{(3.6.2)}$$

If the above requirements are not fulfilled for the longitudinal stiffeners, an alternative design procedure is to replace the yield strength, f_y, with the torsional buckling strength, f_T, in all equations.

$\overline{\lambda_T}$ and f_T may be found in section 3.9.

3.6.3.2 Elastic buckling strength

The elastic buckling strength of longitudinally stiffened cylindrical shells is given by:

$$f_E = C\frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{l}\right)^2 \quad \textbf{(3.6.3)}$$

The reduced buckling coefficient may be calculated as:

$$C = \psi\sqrt{1+\left(\frac{\rho\xi}{\psi}\right)^2} \quad \textbf{(3.6.4)}$$

The values for ψ, ξ and ρ are given in Table 3.6-1 for the most important load cases.

Table 3.6-1 Buckling coefficients for stiffened cylindrical shells, mode b) Panel stiffener buckling

	ψ	ξ	ρ
Axial stress	$\dfrac{1+\alpha_C}{1+\dfrac{A}{s_e t}}$	$0.702\ Z_l$	0.5
Torsion and shear stress	$5.34+1.82\left(\dfrac{l}{s}\right)^{4/3}\alpha_C^{1/3}$	$0.856\ Z_l^{3/4}$	0.6
Lateral Pressure	$2\left(1+\sqrt{1+\alpha_C}\right)$	$1.04\sqrt{Z_l}$	0.6

where

$$Z_l = \frac{l^2}{r\,t}\sqrt{1-\nu^2} \quad \textbf{(3.6.5)}$$

$$\alpha_C = \frac{12\left(1-\nu^2\right)I_{sef}}{s\,t^3} \quad \textbf{(3.6.6)}$$

A = area of one stiffener, exclusive shell plate

I_{sef} = moment of inertia of longitudinal stiffener including effective shell width s_e, see eq. (3.6.7).

3.6.3.3 Effective shell width

The effective shell width, s_e, may be calculated from:

$$\frac{s_e}{s} = \frac{f_{ks}}{\sigma_{j,Sd}}\frac{\left|\sigma_{x,Sd}\right|}{f_y} \quad \textbf{(3.6.7)}$$

where:

f_{ks} = characteristic buckling strength from Section 3.3.2 / 3.4.2.

$\sigma_{j,sd}$ = design equivalent von Mises stress, see eq. (3.2.3).

$\sigma_{x,Sd}$ = design membrane stress from axial force and bending moment, see eq. (2.2.1)

f_y = yield strength

3.7 Orthogonally stiffened shells

3.7.1 General

The buckling modes to be checked are:

a) Shell buckling (unstiffened curved panels), see Sec. 3.7.2
b) Panel stiffener buckling, see Sec. 3.6.
c) Panel ring buckling, see Sec. 3.7.3
d) General buckling, see Sec. 3.7.4
e) Column buckling, see Sec. 3.8

3.7.2 Shell buckling

The characteristic buckling strength is found from Section 3.2 and the elastic buckling strengths are given in Section 3.3.2.

3.7.3 Panel ring buckling

Conservative strength assessment following Section 3.5.2.

3.7.4 General buckling

The rings will normally be proportioned to avoid the general buckling mode. Applicable criteria are given in Section 3.5.

3.8 Column buckling

3.8.1 Stability requirement

The column buckling strength should be assessed if

$$\left(\frac{kL_c}{i_c}\right)^2 \geq 2{,}5\frac{E}{f_y} \quad \textbf{(3.8.1)}$$

where

k = effective length factor
L_C = total cylinder length
i_C = $\sqrt{I_C/A_C}$ = radius of gyration of cylinder section
I_C = moment of inertia of the complete cylinder section (about weakest axis), including longitudinal stiffeners/internal bulkheads if any.
A_C = cross sectional area of complete cylinder section; including longitudinal stiffeners/internal bulkheads if any.

The stability requirement for a shell-column subjected to axial compression, bending, circumferential compression is given by:

$$\frac{\sigma_{a0,Sd}}{f_{kcd}} + \frac{1}{f_{akd}}\left[\left(\frac{\sigma_{m1,Sd}}{1-\frac{\sigma_{a0,Sd}}{f_{E1}}}\right)^2 + \left(\frac{\sigma_{m2,Sd}}{1-\frac{\sigma_{a0,Sd}}{f_{E2}}}\right)^2\right]^{0.5} \le 1 \qquad \textbf{(3.8.2)}$$

where

$\sigma_{a0,Sd}$ = design axial compression stress, see eq. (3.2.4)
$\sigma_{m,Sd}$ = maximum design bending stress about given axis, see eq. (2.2.3)
f_{akd} = design local buckling strength, see Section 3.8.2
f_{kcd} = design column buckling strength, see eq. (3.8.4)
f_{E1}, f_{E2} = Euler buckling strength found from eq. (3.8.3):

$$f_{Ei} = \frac{\pi^2 E I_{c,i}}{\left(k_i L_{c,i}\right)^2 A_c} \quad ,i = 1,2 \qquad \textbf{(3.8.3)}$$

$$f_{kcd} = \frac{f_{kc}}{\gamma_M} \qquad \textbf{(3.8.4)}$$

γ_M = material factor, see eq. (3.1.3)
f_{kc} = characteristic column buckling strength, see eq. (3.8.5) or (3.8.6).

3.8.2 Column buckling strength

The characteristic buckling strength, f_{kc}, for column buckling may be defined as:

$$f_{kc} = [1.0 - 0.28\bar{\lambda}^2] f_{ak} \quad \text{for } \bar{\lambda} \le 1.34 \qquad \textbf{(3.8.5)}$$

$$f_{kc} = \frac{0.9}{\bar{\lambda}^2} f_{ak} \quad \text{for } \bar{\lambda} > 1.34 \qquad \textbf{(3.8.6)}$$

where

$$\bar{\lambda} = \sqrt{\frac{f_{ak}}{f_E}} = \frac{kL_c}{\pi i_c}\sqrt{\frac{f_{ak}}{E}} \qquad \textbf{(3.8.7)}$$

In the general case eq. (3.1.1) shall be satisfied. Hence f_{ak} may be determined (by iteration of equations (3.1.1) to (3.2.6)) as maximum allowable $\sigma_{a0,Sd}$ ($\sigma_{a,Sd}$) where the actual design values for $\sigma_{m,Sd}$, $\sigma_{h,Sd}$ and τ_{Sd} have been applied.

For the special case when the shell is an unstiffened shell the following method may be used to calculate f_{ak}.

$$f_{ak} = \frac{b + \sqrt{b^2 - 4ac}}{2a} \qquad \textbf{(3.8.8)}$$

$$a = 1 + \frac{f_y^2}{f_{Ea}^2} \qquad \textbf{(3.8.9)}$$

$$b = \left(\frac{2 f_y^2}{f_{Ea} f_{Eh}} - 1\right)\sigma_{h,Sd} \qquad \textbf{(3.8.10)}$$

$$c = \sigma_{h,Sd}^2 + \frac{f_y^2 \sigma_{h,sd}^2}{f_{Eh}^2} - f_y^2 \qquad \textbf{(3.8.11)}$$

$$f_{akd} = \frac{f_{ak}}{\gamma_M} \qquad \textbf{(3.8.12)}$$

$\sigma_{h,Sd}$ = design circumferential membrane stress, see eq. (2.2.8) or (2.2.9), tension positive.
f_y = yield strength.
γ_M = material factor, see eq. (3.1.3).
f_{Ea}, f_{Eh} = elastic buckling strengths, see Section 3.4.

3.9 Torsional buckling

The torsional buckling strength may be found from:

- if $\bar{\lambda}_T \le 0.6$:

$$\frac{f_T}{f_y} = 1.0 \qquad \textbf{(3.9.1)}$$

- if $\bar{\lambda}_T > 0.6$:

$$\frac{f_T}{f_y} = \frac{1 + \mu + \bar{\lambda}_T^2 - \sqrt{\left(1 + \mu + \bar{\lambda}_T^2\right)^2 - 4\bar{\lambda}_T^2}}{2\bar{\lambda}_T^2} \qquad \textbf{(3.9.2)}$$

where:

$$\mu = 0.35\left(\bar{\lambda}_T - 0.6\right) \qquad \textbf{(3.9.3)}$$

$$\bar{\lambda}_T = \sqrt{\frac{f_y}{f_{ET}}} \qquad \textbf{(3.9.4)}$$

Generally f_{ET} may be found from:

$$f_{ET} = \beta \frac{G I_t}{I_{po}} + \pi^2 \frac{E h_s^2 I_z}{I_{po} l_T^2} \qquad \textbf{(3.9.5)}$$

For L and T stiffeners f_{ET} may, when eqs. (3.10.4) and (3.10.5) are satisfied, be found from:

$$f_{ET} = \beta \frac{A_W + \left(\frac{t_f}{t_W}\right)^2 A_f}{A_W + 3A_f} G \left(\frac{t_W}{h}\right)^2 + \frac{\pi^2 E I_z}{\left(\frac{A_W}{3} + A_f\right) l_T^2} \qquad \textbf{(3.9.6)}$$

$$I_z = \frac{1}{12}A_f b^2 + e_f^2 \frac{A_f}{1+\frac{A_f}{A_W}} \quad \textbf{(3.9.7)}$$

For flat bar ring stiffeners f_{ET} may be found from:

$$f_{ET} = \left[\beta + 0.2\frac{h}{r}\right] G \left(\frac{t_w}{h}\right)^2 \quad \textbf{(3.9.8)}$$

For flat bar longitudinal stiffeners f_{ET} may be found from:

$$f_{ET} = \left[\beta + 2\left(\frac{h}{l_T}\right)^2\right] G \left(\frac{t_W}{h}\right)^2 \quad \textbf{(3.9.9)}$$

β = 1.0,
or may alternatively be calculated as per eq. (3.9.10)
A_f = cross sectional area of flange
A_W = cross sectional area of web
G = shear modulus
I_{po} = polar moment of inertia = $\int r^2 dA$ where r is measured from the connection between the stiffener and the plate
I_t = stiffener torsional moment of inertia (St. Venant torsion)
I_z = moment of inertia about centroid axis of stiffener normal to the plane of the plate
l_T = for ring stiffeners:
distance (arc length) between tripping brackets.
l_T need not be taken greater than $\pi\sqrt{rh}$ for the analysis;
for longitudinal stiffeners:
distance between ring frames
b = flange width
e_f = flange eccentricity, see Figure 1.3-1
h = web height
h_s = distance from stiffener toe (connection between stiffener and plate) to the shear centre of the stiffener
t = shell thickness
t_f = thickness of flange
t_W = thickness of web

$$\beta = \frac{3C + 0.2}{C + 0.2} \quad \textbf{(3.9.10)}$$

where:

- for longitudinal stiffeners

$$C = \frac{h}{s}\left(\frac{t}{t_w}\right)^3 \sqrt{(1-\eta)}$$

- for ring frames

$$C = \frac{h}{l_{e0}}\left(\frac{t}{t_W}\right)^3 \sqrt{(1-\eta)}$$

and

$$\eta = \frac{\sigma_{j,Sd}}{f_{ks}} \quad \textbf{(3.9.11)}$$

$\sigma_{j,Sd}$ may be found from eq. (3.2.3) and f_{ks} may be calculated from eq. (3.2.1) using the elastic buckling strengths from Sections 3.3.2 or 3.4.2.

Ring frames in a cylindrical shell which is not designed for external lateral pressure shall be so proportioned that the reduced slenderness with respect to torsional buckling, $\bar{\lambda}_T$, is not greater than 0.6.

3.10 Local buckling of longitudinal stiffeners and ring stiffeners

3.10.1 Ring stiffeners

The geometric proportions of ring stiffeners should comply with the requirements given below (see Figure 1.2-1 for definitions):

- Flat bar ring frames:

$$h \le 0.4 t_W \sqrt{\frac{E}{f_y}} \quad \textbf{(3.10.1)}$$

- Flanged ring frames:

$$h \le 1.35 t_W \sqrt{\frac{E}{f_y}} \quad \textbf{(3.10.2)}$$

If the requirements in eqs. (3.10.1) and (3.10.2) are not satisfied, the characteristic material resistance f_r shall be taken as f_T (where f_T is calculated in accordance with Section 3.9).

$$b_f \le 0.4 t_f \sqrt{\frac{E}{f_y}} \quad \textbf{(3.10.3)}$$

where:

b_f = flange outstand

$$\frac{h}{t_w} \le \frac{2}{3}\sqrt{\frac{r_f A_w E}{h A_f f_y}} \quad \textbf{(3.10.4)}$$

$$\frac{e_f}{t_w} \leq \frac{1}{3}\frac{r_f}{h}\frac{A_w}{A_f} \qquad \textbf{(3.10.5)}$$

3.10.2 Longitudinal stiffeners

The geometric proportions of longitudinal stiffeners should comply with the requirements given below (see Figure 1.3-1 for definitions):

- Flat bar longitudinal stiffeners:

$$h \leq 0.4 t_w \sqrt{\frac{E}{f_y}} \qquad \textbf{(3.10.6)}$$

- Flanged longitudinal stiffeners:

$$h \leq 1.35 t_w \sqrt{\frac{E}{f_y}} \qquad \textbf{(3.10.7)}$$

If the requirements in eqs. (3.10.6) and (3.10.7) are not satisfied, the characteristic material resistance f_r shall be taken as f_T (where f_T is calculated in accordance with Section 3.9).

$$h \leq 1.35 t_w \sqrt{\frac{E}{f_y}} \qquad \textbf{(3.10.8)}$$

$$b_f \leq 0.4 t_f \sqrt{\frac{E}{f_y}} \qquad \textbf{(3.10.9)}$$

4. Unstiffened Conical Shells

4.1 Introduction

This chapter treats the buckling of unstiffened conical shells, see Figure 4.1-1.

Buckling of conical shells is treated like buckling of an equivalent circular cylindrical shell.

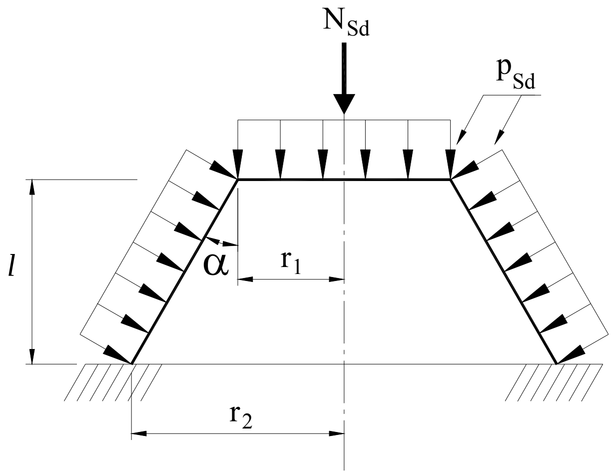


Figure 4.1-1 Conical shell (force and pressure shown is negative)

4.2 Stresses in conical shells

4.2.1 General

The loading condition governing the stresses in a truncated conical shell, Figure 4.1-1, is normally defined by the following quantities:

N_{Sd} = design overall axial force exclusive of end pressure
$M_{1,Sd}$ = design overall bending moment acting about principal axis 1
$M_{2,Sd}$ = design overall bending moment acting about principal axis 2
T_{Sd} = design overall torsional moment
$Q_{1,Sd}$ = design overall shear force acting parallel to principal axis 1
$Q_{2,Sd}$ = design overall shear force acting parallel to principal axis 2
p_{Sd} = design lateral pressure

Any of the above quantities may be a function of the co-ordinate x along the shell generator. In addition p_{Sd} may be a function of the circumferential co-ordinate θ, measured from axis 1. p_{Sd} is always to be taken as the difference between internal and external pressures, i.e. p_{Sd} is taken positive outwards.

The membrane stresses at an arbitrary point of the shell plating, due to any or all of the above seven actions, are completely defined by the following three stress components:

$\sigma_{x,Sd}$ = design membrane stress in the longitudinal direction
$\sigma_{h,Sd}$ = design membrane stress in the circumferential direction
τ_{Sd} = design shear stress tangential to the shell surface (in sections x = constant and θ = constant)

The loading condition and axes are similar as defined for cylindrical shells in Figure 1.1-1.

4.2.2 Longitudinal membrane stress

If simple beam theory is applicable, the longitudinal membrane stress may be taken as:

$$\sigma_{x,Sd} = \sigma_{a,Sd} + \sigma_{m,Sd} \quad \textbf{(4.2.1)}$$

where $\sigma_{a,Sd}$ is due to uniform axial compression and $\sigma_{m,Sd}$ is due to bending.

For a conical shell without stiffeners along the generator:

$$\sigma_{a,Sd} = \frac{p_{Sd}\, r}{2 t_e} + \frac{N_{Sd}}{2\pi r t_e} \quad \textbf{(4.2.2)}$$

$$\sigma_{m,Sd} = \frac{M_{1,Sd}}{\pi r^2 t_e}\sin\theta - \frac{M_{2,Sd}}{\pi r^2 t_e}\cos\theta \quad \textbf{(4.2.3)}$$

where

t_e = t cos α

4.2.3 Circumferential membrane stress

The circumferential membrane stress may be taken as:

$$\sigma_{h,Sd} = \frac{p_{Sd}\, r}{t_e} \quad \textbf{(4.2.4)}$$

where

t_e = t cos α

4.2.4 Shear stress

If simple beam theory is applicable, the membrane shear stress may be taken as:

$$\tau_{Sd} = \left|\tau_{T,Sd} + \tau_{Q,Sd}\right| \quad \textbf{(4.2.5)}$$

where $\tau_{T,Sd}$ is due to the torsional moment and $\tau_{Q,Sd}$ is due to the overall shear forces.

$$\tau_{T,Sd} = \frac{T_{Sd}}{2\pi r^2 t} \quad \textbf{(4.2.6)}$$

$$\tau_{Q,Sd} = -\frac{Q_{1,Sd}}{\pi r t}\cos\theta + \frac{Q_{2,Sd}}{\pi r t}\sin\theta \qquad \textbf{(4.2.7)}$$

where the signs of the torsional moment and the shear forces must be reflected.

4.3 Shell buckling

4.3.1 Buckling strength

The characteristic buckling strength of a conical shell may be determined according to the procedure given for unstiffened cylindrical shells, Section 3.4.

The elastic buckling strength of a conical shell may be taken equal to the elastic buckling resistance of an equivalent unstiffened cylindrical shell defined by:

$$r_e = \frac{r_1 + r_2}{2\cos\alpha} \qquad \textbf{(4.3.1)}$$

$$l_e = \frac{l}{\cos\alpha} \qquad \textbf{(4.3.2)}$$

The buckling strength of conical shells has to comply with the requirements given in Section 3.4 for cylindrical shells. In lieu of more accurate analyses, the requirements are to be satisfied at any point of the conical shell, based on a membrane stress distribution according to Section 4.2.