The Analysis of Stability for the Single Pile under the Bending Moments

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Abstract

The Bending Moments happens when external forces work on many of the Cylindrical Shell structure of marine engineering. Such as the single pile of the offshore wind power, bending moment happens on the top of pile when the axial load and lateral load force on the pile .According to this situation, the cylindrical shell stability formula under axial pressure and the Euler formula are used to form a new stability method of cylindrical shell under bending moment,and the new method is compared with ANSYS, the Russian formula and DNV formula ,ABS formula ,it's accuracy is proved.

Keywords： single pile; cylindrical shell; bending moment; stability.

1 Introduction

Cylindrical shell structure has the advantages of a simple structure and excellent mechanical properties and being manufactured easily, so it is being widely used in marine engineering, ship engineering, bridge engineering, under-water robots and petrochemical projects. With the yield limit of steel continuous improvement, the problem of stability is more and more important to ensure the structural strength. Many researchers are concerned about the stability of the cylindrical shell under bending moment recent years. Some the methods of calculation norms are established, but not enough depth theory. With the increasingly highly request of structure optimization by the various projects field, the cylindrical shell structure under compound external load and the bending moment is applied even more widespread. Therefore, to study the characteristics of stability, not only to further improve the theory of structural mechanics but also has a great significance in engineering design and calculation.

Some of the support structure in the marine projects, as the top must bear the load; there will be a great moment. Calculating the stability of the cylindrical shell under bending moment is very necessary. This article will take the single pile of wind power generation in Bohai Sea for example, and give the calculation method and relevant calculation process of the stability of the single pile under bending moment.

2 The analytical method of stability of cylindrical shell

The single pile used in wind power at sea is a typical cylindrical shell structure. Due to the top of the single pile bear the load and will surely have a moment. Based on classical mechanics, we can solve the problem of the corresponding stress intensity of the cylindrical shell under bending moment, however, the stability of cylindrical shells, there are no precise mathematical models in the absence, making it difficult to get the mathematical analytical solution. Here are a few of the foreign method of calculation.

2.1 The analytical method of DNV Rules

In the References [3], the stability of equation of cylindrical shell under bending moment is this:

$$f_E = C\frac{\pi^2 E}{12(1-\mu^2)}(\frac{t}{l})^2 \tag{2.1}$$

Where

$C = \psi\sqrt{1+(\frac{\rho\varepsilon}{\psi})^2}$, $\psi = 1$,

$\varepsilon = 0.702 Z_l$,

$\rho = 0.5(1+\frac{r}{150t})^{-0.5}$,

$Z_l = \frac{l^2}{rt}\sqrt{1-\mu^2}$,

t—thickness of the shell plate of Cylindrical shell,

l—length of Cylindrical shell,

r—radius of Cylindrical shell,

Owing to $\sigma_{m,Sd} = \frac{M_{1,Sd}}{\pi r^2 t}\sin\theta - \frac{M_{2,Sd}}{\pi r^2 t}\cos\theta$,

Therefore, the moment is:

$$M_E = f_E \times \pi r^2 t = C\frac{\pi^2 E}{12(1-\mu^2)}(\frac{t}{l})^2 \times \pi r^2 t \tag{2.2}$$

2.2 Formula of the actual critical moment in Russia

In the References [7], adopted the basic equations based on The theory of linear come to the critical moment， The results show that the stress amplitude $p_{0,B}$ is over the critical stress only in the percentage when the shell is compressed along the axis ， so the stress' uneven distribution in the section can be ignored in the calculation. Therefore, the critical moment of stress of the shell under bending moment is:

$$p_{0,B} = \hat{p}_{0,B}\frac{Eh}{R} \tag{2.3}$$

The moment is:

$$M_E = p_{0,B} \times \pi r^2 t = \hat{p}_{0,B}\frac{Eh}{R} \times \pi R^2 t \tag{2.4}$$

Where

h—thickness of the shell plate of cylindrical shell,

R—radius of cylindrical shell,

$\hat{p}_{0,B} = 0.6$,

Making accurate experiment for this, the real $\hat{p}_{0,B}$ is 0.6~0.75, and it is a little dispersed.

2.3 The analytical method of ABS rules

In the References [4], the rules of the bending moment for the stability of cylindrical shells is governed axial pressure; as a result, the value of the moment is gained through the stress of the cylindrical shell only under the axial force.

The formula adopted by General Mode:

$$\sigma_e = B\rho_n C\sigma_{cr} \tag{2.5}$$

Where

σ_{cr} =0.605Et/R,

C =1.0, z ≥ 2.85
=1.425/z + 0.175z, z < 2.85

ρ_n =0.75 + 0.003z(1- R /300 t) , z < 1
=0.75 – 0.142 (z-1)0.4+ 0.003z (1- R /300t), 1 ≤ z < 20
=0.35 - 0.0002 R/t , z ≥ 20

B =1.2, λn ≥ 1
=1 + 0.2λn , λn < 1,

λn $= \sqrt{\sigma_y / (\rho_n C\,\sigma_{cr})}$,

$Z = 0.954L^2 / Rt$,

For the theoretical calculation:

$$\sigma_{cr} = 0.605\, E\ t / R \tag{2.6}$$

The moment is:

$$M_E = \sigma_{cr} \times \pi r^2 t = 0.605 Et / R \times \pi R^2 t \tag{2.7}$$

Where

t—thickness of the shell plate of cylindrical shell,

R—radius of cylindrical shell,

3 The new method of cylindrical shell plate buckling analysis

When the cylindrical shells is pressed along axis:

$$F=\frac{\pi R^2}{0.5m^2\alpha^2}\left[\frac{D}{R^3}(n^2-1+m^2\alpha^2)^2+\frac{Et}{R}\frac{m^4\alpha^4}{(m^2\alpha^2+n^2)^2}\right] \quad (3.1)$$

Where $\alpha=\frac{\pi R}{l}$

Slender and long bar under the axial forces, prone to instability Euler buckling:

$$F=\frac{\pi^2 EI}{(\mu l)^2} \quad (3.2)$$

Where

I —Moment of inertia,

l —the length of shell structure,

μ —Length coefficient，

The length of the single pile of the Wind power at sea is relatively long, to some extent, the single pile can be seen as a slender and long bar, taking a small section of the bar as a an object of study, the form of the buckling as figure 2, which is very similar with the form of shell plate buckling as figure 1 showing, just the form of force is different, then we can use the formula (3.2) to solve this problem of buckling of the bar.

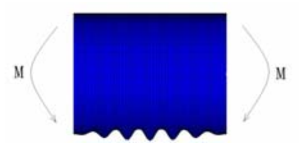

Figure 1 the form of shell plate buckling

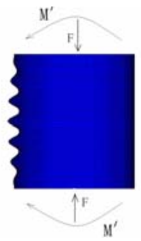

Figure 2 the form of partial bar

If remove the force F in Figure 2, the structure in Figure 1 and the structure in Figure 2 will have the shell will have the same form of force and the same form of buckling. However, just the moment the buckling happen, $M^{/}$ is almost zero, so it can be ignored, therefore, the critical moment of buckling is transformed by the force F . To calculate the critical force of the slender pipe must known the length of slender pipe, So we can combine the formula (3.1) and formula (3.2) to determine the length of the pipe.

$$\frac{\pi R^2}{0.5m^2\alpha^2}\left[\frac{D}{R^3}(n^2-1+m^2\alpha^2)^2+\frac{Et}{R}\frac{m^4\alpha^4}{(m^2\alpha^2+n^2)^2}\right]=\frac{\pi^2 EI}{(\mu l)^2} \quad (3.3)$$

From formula (3.3) we can get the length of the bar, so we can obtain the critical force using the formula (3.2). From the model of shell buckling in Figure 1, we can consider $n=1$ at that time, so the formula (3.3) can be shaped as:

$$\frac{\pi R^2}{0.5}\left[\frac{Dm^2\alpha^2}{R^3}+\frac{Et}{R}\frac{m^2\alpha^2}{(m^2\alpha^2+1)^2}\right]-\frac{\pi^2 EI}{(\mu l)^2}=0 \quad (3.4)$$

l obtain its the maximum value in the calculation, put the $l_{\max}$ into the formula (3.2) to get the $F_{\min}$,and put the $F_{\min}$ into the next formula:

$$\sigma=F\Big/\left[\pi R^2-\pi(R-t)^2\right] \quad (3.5)$$

The moment M can be obtained from formula (3.6) ($D^{/}=2R$)

$$M=\sigma\times W \quad (3.6)$$

where

$$W=\frac{\pi}{32}(1-a^4)$$

$$a=\frac{d}{D^{/}}$$

The new method can be expressed as follows:

$$M=\frac{\pi R^5\left(1-a\right)}{2t\left(2R-t\right)}\cdot\chi \quad (3.7)$$

where

$D^{/}$ —The exterior diameter of a cylindrical shell, mm

d —The inner diameter of a cylindrical shell, mm

where

$$\chi = \frac{Dm^2\alpha^2}{R^3} + \frac{Et}{R}\frac{m^2\alpha^2}{(m^2\alpha^2+1)^2}$$

4 Simplified the new method

In order to make this new method works to facilitate the application, now we simplified the formula (3.7).

Mainly to deal with χ （R =2m, t =0.065m） t/R =0.0325:

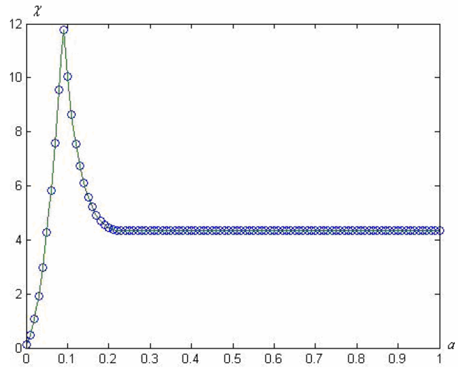

figure 3 the spreading of χ （$0<\alpha<1$）

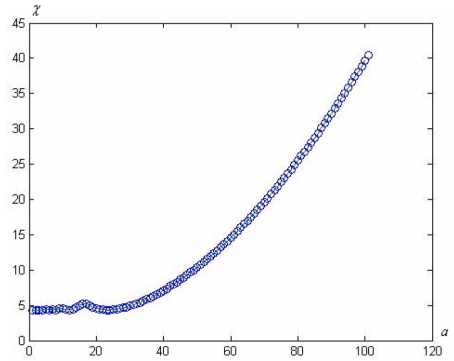

figure 4 the spreading of χ （$\alpha \ge 1$）

From fig 3 and fig 4 χ can be expressed as follow:

$$\chi = \begin{cases} 47.85\alpha \quad 0 \le \alpha < 0.04 \\ 141.72\alpha - 3.3.7549 \quad 0.04 \le \alpha < 0.08 \\ 515.8571\alpha^2 - 168.6714\alpha + 148.1244 \quad 0.08 \le \alpha < 0.19 \\ 4.3499 \quad 0.19 \le \alpha < 13 \\ 0.0047\alpha^2 - 0.1265\alpha + 5.2006 \quad \alpha \ge 13 \end{cases} \tag{4.1}$$

For the cylindrical shell which α is in the range of (0.19,13), α almost have no effect on χ. If α =1, so χ can be simplified as follow:

$$\chi \approx a' m^2 + \frac{b'}{m^2} \tag{4.2}$$

Where

$$a' = D/R^3$$

$$b' = \frac{Et}{R}$$

Because it is hard to determine the m, it is necessary to determine the method of m selection. Get the first derivatives of formula (4.2) and make its value is zero, the m can be expressed as follow:

$$m = \sqrt[4]{\frac{b'}{a'}} \tag{4.3}$$

Put formula (4.3) into the formula (4.2):

$$\chi_{\min} = 2\sqrt{a'b'} \tag{4.4}$$

So the formula (3.7) is simplified at last as follow:

$$M = \kappa M' \quad (0.19 \le \alpha < 13) \tag{4.5}$$

Where

κ —Factor of correction

$$M' = \frac{\pi R^5(1-a)}{2t(2R-t)} \cdot \chi$$

$$\chi = 2\sqrt{a'b'}$$

$$a' = D/R^3$$

$$D = \frac{Et^3}{12(1-\mu^2)}$$

5 The actual example

Take the single pile of wind power in Bohai Sea as an example, the radius of the single pile is R =2 m, thickness of the shell plate is t =0.065 m. Changes the radius and the thickness of the shell plate, and compare the new method with the corresponding analytical solutions.

5.1 Thickness changing

Known radius of the cylindrical shell

R = 2 m, changing the thickness in the range of (0.061 ~ 0.071) m （$\kappa \approx 1$）

thickness (m)	ANSYS (10^9N*m)	Russians (10^9N*m)	DNV (10^9N*m)	ABS (10^9N*m)	New method (10^9N*m)
0.061	2.7880	2.7495	1.2819	2.7724	2.7450
0.063	2.9790	2.9327	1.3712	2.9571	2.9250
0.065	3.1770	3.1219	1.4636	3.1479	3.1106
0.067	3.3810	3.3169	1.5590	3.3446	3.3014
0.069	3.5910	3.5179	1.6575	3.5472	3.4982
0.071	3.8080	3.7248	1.7590	3.7558	3.7003

Table 1 the effect of thickness to buckling

5.2 Radius changing

Known thickness of the cylindrical shell t =0.065 m , changing the radius in the range of (1.7～2.2) m .

radius (m)	ANSYS (10^9N*m)	Russians (10^9N*m)	DNV (10^9N*m)	ABS (10^9N*m)	New method (10^9N*m)
1.7	2.6910	2.6536	1.2603	2.6757	2.6289
1.8	2.8530	2.8097	1.3286	2.8331	2.7894
1.9	3.0150	2.9658	1.3964	2.9905	2.9500
2.0	3.1770	3.1219	1.4636	3.1479	3.1106
2.1	3.3390	3.2780	1.5303	3.3053	3.2712
2.2	3.5010	3.4340	1.5964	3.4627	3.4318

Table 2 the effect of radius to buckling

From table 1 and table 2, we can see that results of new method is very closer with the results of Russian formula, ABS roles, and the software ANSYS, but it is very difference from the results of the DNV roles, so it can see get the conclusion that the DNV roles are too conservative in somewhat, but it also can prove that the new method is reasonable.

6 Conclusions

(1) By combining the formula of cylindrical shell buckling and the formula of bar buckling, deriving a new method of thin-walled shell buckling under the bending moment.

(2) Compared with the Russia recommended formula, ABS rules, DNV rules, and the ANSYS software, it proves that the new formula has high accuracy, and it can be used in Practical application.

(3) In order to facilitate the engineering application, the formula was simplified reasonably, and give the scope of this formula's application.

(4) It is show that the calculating results of DNV rules are not very ideal, so it need to be further analyzed. And the ABS rules and the Russia rules so precise that it can be used in the calculation of shell buckling. For some complex structure, we can use the software of finite element to establish model and calculate the model.

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