Three-dimensional dynamic analysis of jack-up structures

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ABSTRACT: As mobile jack-up drilling rigs continue to move into deeper waters and harsher environments there is an increased need to understand their behaviour under storm loading conditions. To improve the assessment of jack-ups for a specific site it has become necessary to analyse these units in three dimensions with models that appropriately reflect the physical processes occurring. This motivated the development of a computer program (SOS_FE3D) that takes a balanced approach to all three inter-related components of the structure, the foundations and the environmental loading in three dimensions. Geometrical structural nonlinearities are incorporated using a path-dependent formulation of beam-column theory to specify an incremental stiffness matrix. The program implements a six-degree of freedom strain-hardening plasticity model to simulate the soil-structure interaction and, when fully developed, advanced formulations for environmental loads. In this paper, the results of three-dimensional jack-up analyses are presented and compared with two-dimensional simulations, and the importance of dynamic assessments will also be highlighted.

1 INTRODUCTION

Mobile jack-up drilling rigs (see Fig. 1) are widely used in offshore oil and gas exploration because they offer economic advantages such as comparably simple and quick installation. However, they do suffer from a higher accident rate than traditional fixed platforms. Further, as the trend continues to employ jack-up rigs for year-round drilling in deeper water and harsher environments, there is an increased need to understand their behaviour under realistic loading conditions. This will allow the capacity of the platform to be more accurately assessed and the accident rate reduced.

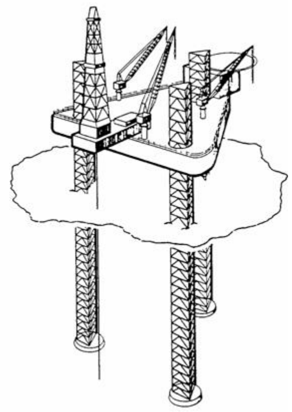


Figure 1. Typical jack-up unit (after Williams et al. (1998)).

During installation and in a perfectly calm sea, the vertical self-weight of an offshore structure is the dominant loading on its footings. During a storm, however, environmental wind and wave forces impose additional horizontal loads, overturning moments and even torsion loads on the foundations, as well as altering the sharing of vertical load among the footings. This combined loading of a foundation results in a complex state of stress and strain in the underlying soil.

Accurate modelling of the interaction between the structure and foundation is important for the assessment of dynamically sensitive structures, such as jack-ups, as their response depends significantly on the combined stiffness of the structure/foundation system.

This paper introduces the computer program *SOS_FE3D* for the **S**imulation of **O**ffshore **S**tructures complete with their **F**oundations and **E**nvironmental loading in **3 D**imensions. It is a finite element program which, when fully developed, will be capable of analysing the soil-structure-fluid interaction of offshore structures in three dimensions. Its first application will be for the analysis of jack-ups and their shallow spudcan foundations.

Only the structural and foundation models are described in this paper. An example analysis of a more traditional plane-frame two-dimensional model is initially described and the additional advantages of the three dimensional options are then shown. The importance of dynamic simulations using advanced foundation models is also highlighted. Current research is concentrating on the development of improved three-dimensional wave loading models and cyclic loading foundation models and although no results are shown here, a discussion of these is given.

2 THE ANALYSIS PROGRAM *SOS_FE3D*

2.1 *The structural model*

As tall slender structures (such as jack-ups or wind-turbines) experience significant deformations with respect to their overall dimension, small deformation analysis based on equilibrium of the undeformed structure is not applicable. In order to obtain accurate results, the analysis must account for the deformed configuration of the system.

As the computer program *SOS_FE3D* aims at these applications, the structure is discretised as a frame built of beam-columns and the numerical procedure for large deformation analysis is used. The procedures adopted here are outlined in Kassimali (1983) for plane problems and Kassimali & Abbasnia (1991) for space structures. They were chosen as they contain no assumptions beyond those already inherent in conventional beam-column theory. Kassimali's method caters for frames composed of prismatic members, with loads applied at the joints. Loads are assumed to move with their respective joints as the structure deforms. The procedure is based on an Eularian formulation. This method clearly separates the contribution of rigid body displacements, which may be arbitrarily large, from relative member deformations, which are considered to be small enough to justify the use of conventional beam-column theory.

This formulation inherently accounts for P-Δ effects due to axial forces no longer acting along the centroid of the beam. Changes in member chord length due to axial strains and flexural bowing and the influence of axial force on member flexural stiffness are taken into account. This is particularly important in dynamic analyses. In earlier work by Oran (1973), the method is shown to be highly accurate, even in the presence of substantial deflections.

In the dynamic evaluation of nonlinear structural responses, an explicit damping matrix is required and in *SOS_FE3D*, Rayleigh damping is employed. This method uses a linear combination of stiffness and mass proportional damping as shown in Equation (1).

$$[C] = a_0[M] + a_1[K] \qquad (1)$$

where $[C]$ is the damping matrix, $[M]$ the mass matrix, $[K]$ the stiffness matrix of the system and a_0 and a_1 the Rayleigh damping factors. These factors can be evaluated by the solution of a pair of simultaneous equations, with the damping ratios associated with two specific modes specified by the user. The consistent mass matrix formulation is used (see for instance Przemieniecki (1968)).

Material nonlinearity is not considered in *SOS_FE3D*.

2.2 *The foundation model*

A major aim in the development of *SOS_FE3D* is the inclusion of sophisticated force-resultant models describing non-linear soil-structure interaction. In the analysis of jack-ups incorporation of the load-displacement behaviour of the large spudcan footings is required to accurately predict the response of the system. Footing assumptions such as pinned restraints or springs oversimplify the foundation behaviour and may even lead to unconservative results. This has been shown in two-dimensional dynamic analyses by, amongst others, Williams et al. (1998).

Force-resultant models, which formulate the soil-foundation interaction as a 'macro element' in terms of strain-hardening plasticity theory, have been shown to model the nonlinear response displayed in experimental tests well (Cassidy et al. (2003), Houlsby (2003)) and have also been applied to simulate monitored offshore jack-up data (Cassidy et al. (2002)). These models can be easily implemented into conventional analysis programs, with the advantage that the complete soil-footing behaviour can be incorporated directly as 'point' element attached to the node of a structural element. This eliminates any need for special interface elements between the structure and the soil.

The numerical footing model implemented in *SOS_FE3D* is called *ISIS* and features options to model flat circular footings, cones, spudcans or suction caissons on both clay or sand. The ISIS models are formulated within a strain-hardening plasticity framework in load-displacement space for six degrees of freedom (as shown in Fig. 2).

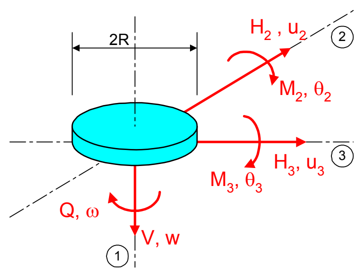


Figure 2. Sign convention for the *ISIS* footing model

Detailed information on the plasticity-based footing model for clay may be found in Martin (1994), Martin & Houlsby (1999) and Martin & Houlsby (2001). Publications on the respective model for footings on sand include Cassidy et al. (2002), Byrne & Houlsby (2001) for uncemented loose carbonate sand and Houlsby & Cassidy (2002), Cassidy & Bienen (2002) and Cassidy (1999) for dense silica sand.

Plasticity models feature four main components: a yield surface, hardening law, flow rule and an elastic representation.

The yield surface has been established through experimental investigation and is written as

$$f = 0 = \left(\frac{H_3}{h_0 V_0}\right)^2 + \left(\frac{M_2/2R}{m_0 V_0}\right)^2 - \frac{2aH_3 M_2/2R}{h_0 m_0 V_0^2} + \left(\frac{H_2}{h_0 V_0}\right)^2 + \left(\frac{M_3/2R}{m_0 V_0}\right)^2 - \frac{2aH_2 M_3/2R}{h_0 m_0 V_0^2} + \left(\frac{Q/2R}{q_0 V_0}\right)^2 - \left[\frac{(\beta_1 + \beta_2)^{(\beta_1+\beta_2)}}{\beta_1^{\beta_1}\beta_2^{\beta_2}}\right]^2 \cdot \left(\frac{V}{V_0}\right)^{2\beta_1} \cdot \left(1 - \frac{V}{V_0}\right)^{2\beta_2} \quad (2)$$

where V_0 is the vertical load capacity and R the radius of the footing. Sections of the yield surface (shown for the two dimensional case in Fig. 2) including the V-axis are approximately parabolic, and sections normal to the V-axis are approximately elliptical. The dimensions of the yield surface are determined by h_0 and m_0, respectively, and a accounts for eccentricity of the elliptical yield surface in the $M_2/2R$: H_3 and $M_3/2R$: H_2 planes. The parameters β_1 and β_2 round off the ends of the yield surface in order to avoid numerical difficulties.

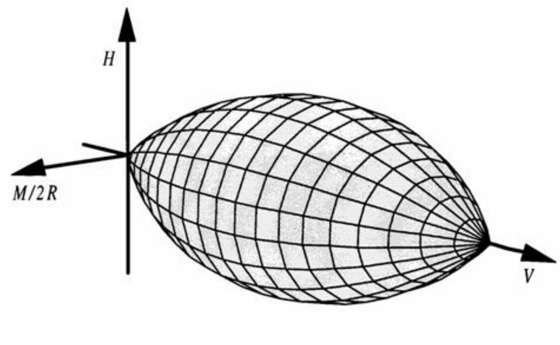


Figure 2. Yield surface (V, H, M/2R) (Cassidy (1999)).

Once the yield surface is established, any changes of load within this surface ($f < 0$) are assumed to be entirely elastic and are described as

$$\begin{bmatrix} dV \\ dH_2 \\ dH_3 \\ dQ \\ dM_2 \\ dM_3 \end{bmatrix} = \begin{bmatrix} K_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_3 & 0 & 0 & 0 & -K_4 \\ 0 & 0 & K_3 & 0 & K_4 & 0 \\ 0 & 0 & 0 & K_5 & 0 & 0 \\ 0 & 0 & K_4 & 0 & K_2 & 0 \\ 0 & -K_4 & 0 & 0 & 0 & K_2 \end{bmatrix} \begin{bmatrix} dw^e \\ du_2^e \\ du_3^e \\ d\omega^e \\ d\theta_2^e \\ d\theta_3^e \end{bmatrix} \quad (3)$$

where K_1, K_2, K_3, K_4 and K_5 are elastic constants (a combination of the shear modulus, radius and non-dimensional elastic factors determined through finite-element analysis), and w^e, u^e, ω^e and θ^e represent the elastic vertical, horizontal and rotational displacements, respectively.

When the load state touches the yield surface ($f = 0$) it is usual for the foundation to penetrate further into the soil, causing plastic deformations to occur and the capacity of the foundation to increase (expansion of the yield surface). The hardening law links the vertical plastic displacement with vertical load capacity V_0 and therefore defines how the size of the yield surface varies with penetration. Although the yield surface size may vary, its shape remains essentially constant as reflected in Equation 2.

The ratios between the plastic footing displacement components during yield are predicted by the flow rule.

In a numerical plasticity-based model of a shallow foundation such as a spudcan or suction caisson used offshore, the loading is applied incrementally, and the force resultant model computes an updated tangent stiffness matrix for each footing for each step. These stiffness matrices are then incorporated into the structural stiffness matrix at the respective degrees of freedom. Using this system stiffness matrix the response increment is evaluated. The calculated deformations at the footing node are in turn passed on to the numerical footing model. At this stage the updated load state of the footing as well as the plastic deformation components are found.

2.3 *Solution method*

The analysis options available in *SOS_FE3D* are:

- linear or geometrically nonlinear structure,
- quasistatic or dynamic analysis,
- in two or three dimensions, and
- pinned, fully-fixed or ISIS model footings.

In a finite element representation of a structure the problem may be formulated as

$$[M]\{\Delta\ddot{x}\} + [C]\{\Delta\dot{x}\} + [K]\{\Delta x\} = \{\Delta P\} \quad (4)$$

where $\{\Delta\ddot{x}\}$ is the change in acceleration, $\{\Delta\dot{x}\}$ the change in velocity, $\{\Delta x\}$ the change in displacement and $\{\Delta P\}$ the increment of external load applied.

In a quasistatic analysis, Equation (4) reduces to the incremental equilibrium equation given by Equation (5) as both the velocity and the acceleration are zero.

$$[K]\{\Delta x\} = \{\Delta P\} \quad (5)$$

In a linear analysis where equilibrium is established on the undeformed configuration, the stiffness matrix remains unchanged throughout the solution process and Equation (5) can be solved for the change in displacement $\{\Delta x\}$ for each time increment. The total displacement over the duration of the analysis is the sum of all $\{\Delta x\}$ components. No iteration is required.

The solution algorithm used for the nonlinear analyses is Newton-Raphson. Iteration is performed at each load level until the correction displacement vector is sufficiently small to deem the equilibrium satisfied.

For a dynamic analysis of a nonlinear system, where the eigensystem is constantly changing, direct integration methods are well suited because they avoid any use of superposition. The loading history is divided into a sequence of time increments, and Equation (4) is solved using a numerical step-by-step procedure.

In order to carry out this type of analysis, it is first necessary to assume how the acceleration varies during the time step. The change of velocity over the time increment depends on the integral of the acceleration, and the change of displacement depends on the corresponding velocity integral. The values of these quantities at the end of the time step are composed of the addition of their initial values at the beginning of the step and their change over the time increment. Therefore, the response for each step is an independent analysis problem. The step-by-step methods are equally valuable in the analysis of linear response as the same algorithm can be applied.

here are many different step-by-step methods. The integration method implemented in *SOS_FE3D* is the implicit, unconditionally stable 'Newmark constant average acceleration' (or Newmark $\beta = 1/4$). As the name implies, the acceleration is assumed to be constant over the time increment. Therefore, the velocity varies linearly and the displacement in a quadratic manner.

For a linear system, the new displacement and velocity can be evaluated directly using the acceleration at the end of the previous step as the stiffness, mass or damping matrices do not change with time. The solution is not improved by iteration during a time step, but may be improved by a reduction in the time step size.

For a nonlinear system, a combination of both a reduction in the size of the time step and iteration dictates the accuracy.

3 EXAMPLE SIMULATIONS

The benefits and versatility of *SOS_FE3D* will be illustrated with example jack-up simulations. These include plane-frame analyses in two- and three-dimensions and simulations of a full three-dimensional jack-up model.

3.1 *Starting point – plane frame in 2D*

For computational ease jack-ups are often idealised as two-dimensional plane-frame structures, a condition allowed for in their site-specific assessment (SNAME (1997)). Further, as nearly all soil-structure interaction models have only been formulated for two dimensional analyses this idealisation has continued. Any environmental (horizontal) loading is assumed to act along the "axis of symmetry" of the hull, only allowing two different loading directions: one leg windward / two legs leeward or two legs windward / one leg leeward.

A 2D plane-frame analysis will be used as the starting point and comparison will be made to this response to demonstrate correct implementation of the three-dimensional formulation.

Shown in Figure 3 is the plane frame configuration analysed (the former of the two cases), with the structural properties of the model given in Table 1 and those of the spudcan and clay in Table 2. For all analyses in this paper the damping ratios of the lowest two modes have been set at 5%. Further, self-weight (W) and environmental loads (H_{env}) are applied as point loads at the leg/hull interface.

Table 1. Properties. of the jack-up (plane frame).

Preload per spudcan	100 MN	Self-weight per spudcan	50 MN
Young's modulus E	200 GPa	Shear modulus G	80 GPa
I_{leg}	10.843 m^4	I_{hull}	50.0 m^4
A_{leg}	0.6 m^2	A_{hull}	2.0 m^2
$Mass_{leg}$	1.93E6 kg	$Mass_{hull}$	16.1E6 kg

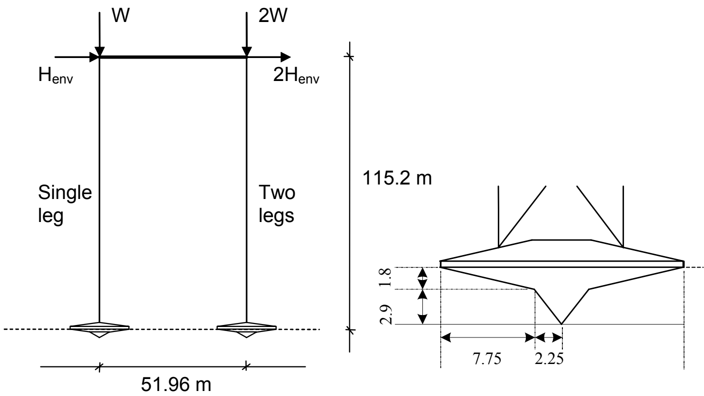


Figure 3. 2D model jack-up and spudcan (all dimensions in m)

Table 2. Spudcan and clay properties.

Spudcan shape	76° / 154° / 130°
Spudcan roughness α	1.0
Submerged unit weight γ'	10 kN/m3
Mudline strength	10 kPa
Increase with depth ρ	2.0 kPa/m
Rigidity index Ir	100

In the analysis the jack-up is initialized with a vertical preload of twice the self-weight (common field installation practice). This is numerically performed before the time-stepping procedure begins. The jack-up is then brought to its self-weight (time 0→100s), with the combined load state of each footing lying in the centre of the yield surface. Horizontal load is then applied in two stages (100→200s and 300→400s), between which it is held constant. At 700s all of the horizontal loads are instantaneously removed.

In Figure 4, the horizontal hull displacement due to this loading history is shown. Elastic behaviour was observed during the first loading stage as the combined loading state of all the footings stayed within their respective yield surfaces. However, as the applied horizontal load increases further the combined spudcan load-state eventually touches and expands the yield surface. This causes yielding of the soil and a non-linear degradation of stiffness to occur, as indicated by the kink in the curve at t ≈ 360 s (*ISIS* footing analysis). Significant dynamic effects are observed due to the sudden load loss at 700s, and these results can be used to confirm level of damping. The results also show a permanent horizontal offset of the hull due to the yielding of the soil beneath the spudcans.

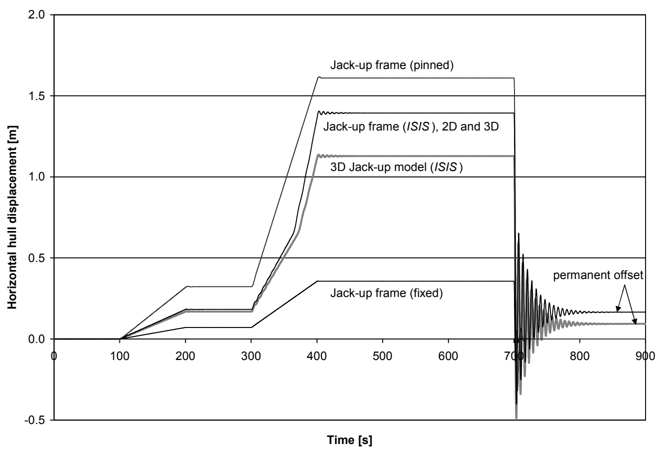


Figure 4. Horizontal hull displacement jack-up frame (2D/3D).

Comparisons to more traditional foundation assumptions of pinned and fixed footings are also shown in Figure 4. These highlight the influence of footing stiffness on the system response. At the beginning of the analysis, the response of the frame with *ISIS* footings lies approximately halfway between these footing assumptions. However, as the footings yield the response approaches the pinned case. Furthermore, with these simple footing assumptions there is no permanent offset of the hull.

Accurate modelling of the footing restraints is paramount as the natural period of the system increases with decreasing stiffness. The jack-up's natural period is 8.3 and 4.1 s for the pinned and fixed cases, respectively. The natural period for the *ISIS* model case was 6.4 s before yield and increased towards the pinned value during yielding. This becomes important during realistic storm loading conditions as the dominant loading period for waves is in the order of 8-14s.

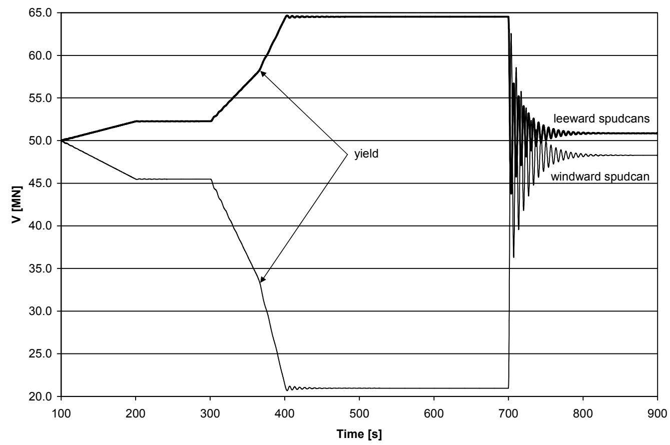


Figure 5. Vertical spudcan response (plane frame).

The load sharing of the spudcans is shown in Figure 5, indicating that due to the overturning load the leeward footings take relatively more vertical load. Also identifiable in this plot are the yield points.

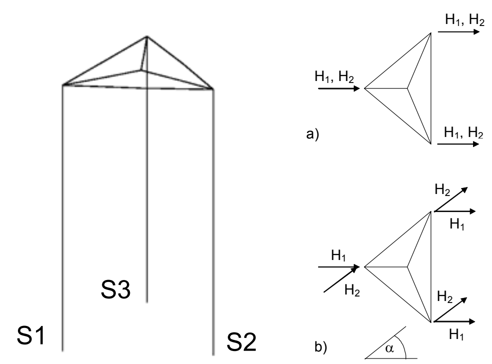
3.2 *Check on implementation – plane frame in 3D*

The same plane frame and loading conditions have also been analysed in three dimensions. As expected, the results match exactly, which shows that the three-dimensional model formulations and the solution methods have been implemented successfully.

3.3 *3D discretization of a jack-up*

In order to complete the transition from two- to three-dimensional modelling, a full three-dimensional jack-up is discretised and is shown in Figure 6. The model has been kept simple in order to highlight the computational advances in *SOS_FE3D* and to minimize computational effort. The properties are similar to those used for the plane frame (Tables 1 and 2). The total mass of the hull is the same, but now distributed evenly across the beam grillage.

Figure 6. 3D model jack-up (structure only).



For the same loading history (Fig. 6a) the response of the 3D model jack-up is included in Figure 4. The three-dimensional modelling of the hull alters the structural response of the jack-up slightly as the grillage of beams cannot exactly match the constant structural properties of the one beam element of the plane frame. Because of the resulting different load transfer through the structure, the reaction at the soil/structure interface is altered slightly, too. As a result, the soil yields less, which in turn leads to less (permanent) deformations.

Nevertheless, the responses match well and the transition form two-dimensional plane frame analysis to full three-dimensional analysis has been successful.

3.4 *Influence of the loading direction*

Now the direction of loading on the 3D jack-up model is altered at the second stage. This is to show the influence of non-collinear loading conditions and could be thought to represent wave and current loading in one orientation with the wind direction changing. In the analyses the load magnitudes of the two stages have been kept as before. However, the direction of the second stage has been altered from collinear (Fig. 6a) to angles of 45 and 90° (Fig. 6b).

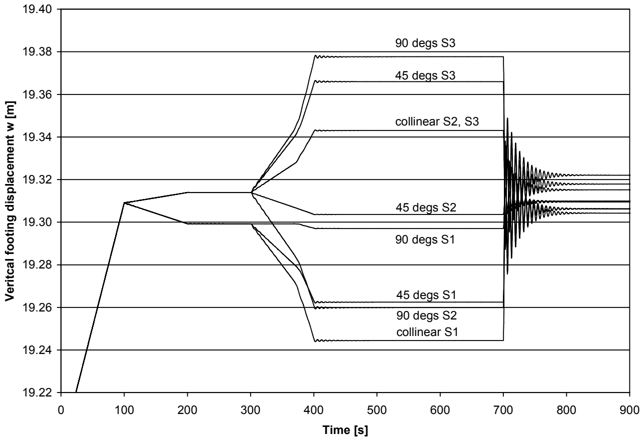


Figure 7. Loading situations (3D)

As an example Figure 7 shows the vertical displacements of the footings. Of course, all load and displacement responses are being evaluated and could be interpreted. Again the non-linear behaviour due to the footing plasticity can be observed in the second stage and this stresses the importance of a sophisticated load-displacement type of foundation model. Further, with these changes in stiffness the reactions are significantly altered during the analysis influencing the overall system response. Capturing these changes would not be possible without accurate modeling of the soil-structure interaction in all six-degrees of freedom. The advantage of three-dimensional modelling are also highlighted as now any combination of loads in any loading direction can be prescribed.

4 CONCLUSION

In this paper, the computer program *SOS_FE3D* for the simulation of offshore structures has been introduced. Through use of example analyses of a jack-up rig the importance of three-dimensional dynamic modelling using a sophisticated load-displacement relationship to model the soil-structure interaction has been highlighted. Although employed for jack-up analyses here, the program may also be used to simulate other structures composed of beam-columns. With the advantage of including force-resultant models that describe shallow foundation behaviour, example applications include offshore wind turbines and pipelines laying untrenched on the sea-bed.

Further developments of the program are ongoing. These include implementation of wave loading models in three-dimensions that will allow investigation of response to random spread seas, and development of a more sophisticated foundation model that accounts for cyclic loading of shallow foundations.

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REFERENCES

Byrne, B.W. & Houlsby, G.T. 2001. Observations of Footing Behaviour on Loose Carbonate Sands. *Géotechnique.* Vol. 51 (5): 463-466.

Cassidy, M.J. 1999. Non-Linear Analysis of Jack-Up Structures Subjected to Random Waves. PhD thesis. *University of Oxford*.

Cassidy, M.J. & Bienen, B. 2002. Three-Dimensional Numerical Analysis of Jack-Up Structures on Sand. *Proc. 12th International Offshore and Polar Engineering Conference*.

Cassidy, M.J., Byrne, B.W. & Houlsby, G.T. 2002. Modelling the behaviour of circular footings under combined loading on loose carbonate sand. *Géotechnique*. 52 (10): 705-712.

Cassidy, M.J., Houlsby, G.T., Hoyle, M. & Marcom, M. 2002. Determining appropriate stiffness levels for spudcan foundations using jack-up case records. *Proc. 21st Int. Conf. On Offshore Mechanics and Arctic Engineering (OMAE)*. OMAE2002-28085.

Cassidy, M.J., Martin, C.M. & Houlsby, G.T. 2003. Development and application of force resultant models describing jack-up foundation behaviour. *Proc. 9th Int. Conf. on Jack-up Platform Design, Construction and Operation*.

Houlsby, G.T. & Cassidy, M.J. 2002. A plasticity model for the behaviour of footings on sand under combined loading. *Géotechnique.* 52 (2): 117-129.

Houlsby, G.T. 2003. Modelling of shallow foundations for offshore structures. *International Conference on Foundations*.

Kassimali, A. 1983. Large deformation analysis of elasto-plastic frames. *Journal of Structural Engineering, ASCE.* 109 (8): 1869-1886.

Kassimali, A. & Abbasnia, R. 1991. Large deformation analysis of elastic space frames. *Journal of Structural Engineering, ASCE.* 117 (7): 2069-2087.

Martin, C.M. 1994. Physical and Numerical Modelling of Offshore Foundations Under Combined Loads. PhD thesis. *University of Oxford*.

Martin, C.M. & Houlsby, G.T. 1999. Jackup units on clay: structural analysis with realistic modelling of spudcan behaviour. *Offshore Technology Conference.* OTC 10996.

Martin, C.M. & Houlsby, G.T. 2001. Combined loading of spudcan foundations on clay: numerical modelling. *Géotechnique*. 51 (8): 687-699.

Oran, C. 1973. Tangent stiffness in plane frames. *Journal of the Structural Division, ASCE*. 99 (ST6): 973-985.

Przemieniecki, J.S. 1968. *Theory of matrix structural analysis*. McGraw-Hill.

SNAME T&R 5-5A. 1997. Site specific assessment of mobile jack-up units. 1st Edition – Rev 1. *Society of Naval Architects and Marine Engineers*, New Jersey.

Williams, M.S., Thompson, R.S.G. & Houlsby, G.T. 1998. Non-linear dynamic analysis of offshore jack-up units. *Computers and Structures*. 69: 171-180.