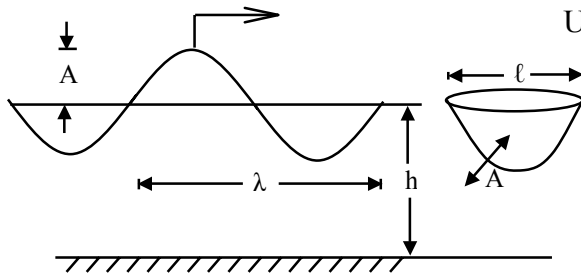


# 13.021 – Marine Hydrodynamics Lecture 23

## Wave Forces on a Body



$$C_F = \frac{F}{\rho g A \ell^2} = f\left(\frac{A}{\lambda}, \frac{\ell}{\lambda}, R, \frac{h}{\lambda}, \text{roughness}, \dots\right)$$

Wave steepness

Diffraction parameter

$U = \omega A$  body velocity, particle velocity

$$R = \frac{U \ell}{v} = \frac{\omega A \ell}{v}$$

$$K_c = \frac{UT}{\ell} = \frac{A \omega T}{\ell} = 2\pi \frac{A}{\ell}$$

## Types of Forces

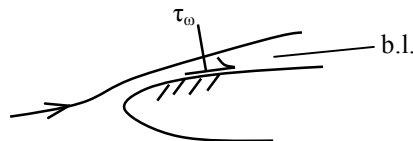
(1) Viscous forces  $f(R, K_c, \text{roughness}, \dots)$

(a) Form drag – form associated primarily with flow separation



(b) Friction drag

$$F \sim \iint_{\text{body}} \tau_w dS$$



(2) Inertial forces – forces arising from potential flow wave theory

$$\bar{F} = \iint_{\text{body (wetted surface)}} p \hat{n} dS$$

$$p = -\rho \left( \frac{\partial \phi}{\partial t} + gy + \frac{1}{2} |\nabla \phi|^2 \right)$$

If linear theory  
Small amplitude waves

In general:  $\phi = \phi_I + \phi_D + \phi_R$  → Radiated wave potential

Incident wave potential

Diffacted wave potential

$$p = -\rho \left( \frac{\partial \phi_I}{\partial t} + \frac{\partial \phi_D}{\partial t} + \frac{\partial \phi_R}{\partial t} + gy + \dots \right)$$

(a) (b) (c)

## (2) (a) Froude-Krylov Force

When  $\ell \ll \lambda$ , incident wave field is not significantly modified by the presence of the body, therefore ignore  $\phi_D, \phi_R$  :

$$\phi \approx \phi_I$$

$$\bar{F}_{FK} = \underbrace{\iint_{\text{body surface}} -\rho \left( \underbrace{\frac{\partial \phi_I}{\partial t} + gy}_{p_I} \right) \hat{n} dS}_{\text{can calculate knowing (incident) wave kinematics (and body geometry)}}$$

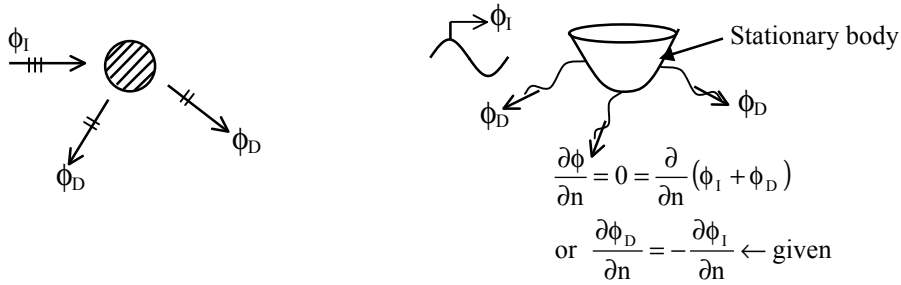
If body is really small, the integral can be replaced by

$$\bar{F}_{FK} = - \iiint_{\substack{\text{body} \\ \text{volume}}} \nabla p_I d\forall \approx - \nabla p_I \Big|_{\substack{\text{body} \\ \text{center}}} \star \forall \quad \begin{matrix} \text{Body} \\ \text{volume} \end{matrix}$$

(b) Radiation and Diffraction Forces - hydrodynamic coefficients: added mass, wave damping and wave excitation

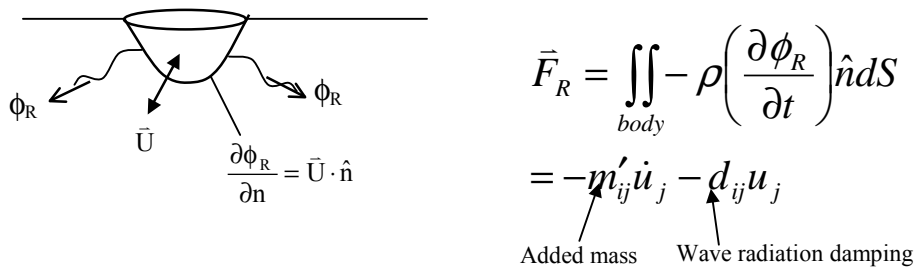
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(b.1) Diffraction or scattering force: when  $\ell$  not  $\ll \lambda$ , wave field near body will be affected even if body is stationary.



$$\bar{F}_D = \iint_{\text{body}} -\rho \left( \frac{\partial \phi_D}{\partial t} \right) \hat{n} dS$$

(b.2) Radiation Force – added mass and damping coefficient:  
even in the absence of an incident wave, body in motion creates waves and hence wave forces, and experiences also inertial forces.



## 2) Important Parameters

$$\left. \begin{array}{l} (1) K_c = \frac{UT}{\ell} = 2\pi \left( \frac{A}{\ell} \right) \\ (2) \text{ diffraction parameters } \frac{\ell}{\lambda} \end{array} \right\} \begin{array}{l} \text{interrelated since maximum wave steepness} \\ \frac{A}{\lambda} \leq 0.07 \\ \left( \frac{A}{\ell} \right) \left( \frac{\ell}{\lambda} \right) \leq 0.07 \end{array}$$

→ if  $K_c \leq 1$ : no appreciable flow separation, viscous effect confined to b.l. (hence small), solve problem via potential flow theory.

→ if  $\frac{\ell}{\lambda} \ll 1$  ignore diffraction, wave effects in radiation problem

(i.e.  $d_{ij} \approx 0$ ,  $m'_{ij} \approx m'_{ij}$  infinite fluid added mass)

calculate  $\bar{F}_{FK}$

→ if  $\frac{\ell}{\lambda} \geq \frac{1}{5}$  must consider wave diffraction, radiation  $\left\{ \frac{A}{\ell} \lesssim \frac{0.07}{\ell/\lambda} \lesssim 0.35 \right\}$

→ if  $K_c \ll 1$ : separation important, viscous force dominates

$$\frac{\ell}{\lambda} \leq \frac{0.07}{A/\ell} \quad \text{so} \quad \frac{\ell}{\lambda} \ll 1 \quad \text{ignore diffraction}$$

$$F = \frac{1}{2} \rho \ell^2 \underset{\substack{\uparrow \\ \text{Relative velocity}}}{U(t)} |U(t)| C_D(R)$$

→ Intermediate  $K_c$  – both viscous and inertial effects important  
use Morison's formula

$$F = \frac{1}{2} \rho \ell^2 U |U| C_D(R, K_c) + \rho \ell^3 \dot{U} C_m(R, K_c)$$

